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The stationary non-equilibrium plasma of cosmic-ray electrons and positrons

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HIGHLIGHTS

- The thermodynamic variables of the cosmic-ray electron–positron plasma are calculated.
- Spectral fits to the AMS-02 and HESS GeV–TeV electron and positron fluxes are performed.
- A semi-empirical phase-space reconstruction of the partial probability densities is carried out.
- Partition function & entropy of a relativistic plasma in stationary non-equilibrium are derived.
- The positron fraction extrapolated to TeV energies shows a broad peak & exponential decay.

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ABSTRACT

The statistical properties of the two-component plasma of cosmic-ray electrons and positrons measured by the AMS-02 experiment on the International Space Station and the HESS array of imaging atmospheric Cherenkov telescopes are analyzed. Stationary non-equilibrium distributions defining the relativistic electron–positron plasma are derived semi-empirically by performing spectral fits to the flux data and reconstructing the spectral number densities of the electronic and positronic components in phase space. These distributions are relativistic power-law densities with exponential cutoff, admitting an extensive entropy variable and converging to the Maxwell–Boltzmann or Fermi–Dirac distributions in the non-relativistic limit. Cosmic-ray electrons and positrons constitute a classical (low-density high-temperature) plasma due to the low fugacity in the quantized partition function. The positron fraction is assembled from the flux densities inferred from least-squares fits to the electron and positron spectra and is subjected to test by comparing with the AMS-02 flux ratio measured in the GeV interval. The calculated positron fraction extends to TeV energies, predicting a broad spectral peak at about 1 TeV followed by exponential decay.

1. Introduction

We study the statistical mechanics of the cosmic-ray electron–positron plasma, based on high-precision spectra obtained with the Alpha Magnetic Spectrometer (AMS-02) [1,2]. We demonstrate that this plasma can be treated, in close analogy to the photon gas of the cosmic microwave background radiation, as a primordial electron gas in stationary non-equilibrium. A primordial origin is also suggested by the high isotropy of the observed fluxes. Primordiality of the cosmic-ray electron–positron plasma is tantamount to the material realization of a universal cosmic reference frame as the rest frame of a relativistic gas of massive particles. An alternative approach is to use relativistic kinetic theory, but to arrive at

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quantitative densities suitable for spectral fitting, one has to specify production mechanisms and interaction processes with cosmic matter and radiation fields which are uncertain [3].

We perform spectral fits to the AMS-02 electron and positron fluxes and reconstruct, from the inferred spectral densities, the distribution functions of the electronic and positronic plasma components. The partial number densities are relativistic non-equilibrium distributions, exponentially cut power-law densities [4,5] which converge to the Maxwell–Boltzmann distribution for low particle velocities, the Coulomb interaction being negligible. We calculate the classical partition function, by ensemble averaging in phase space, and the entropy function which is an extensive variable. We then quantize the grand partition function and show, based on the fugacity obtained from the spectral fits, that the classical limit is realized. Finally we calculate the positron fraction and give estimates of the thermodynamic parameters of this low-density high-temperature plasma.

In Section 2, we consider a relativistic plasma in stationary non-equilibrium and relate the classical spectral number density to the empirical flux density obtained from spectral fits to the measured electron and positron fluxes (performed in Section 5). In Section 3, we derive the thermodynamic variables of the electronic and positronic plasma components in phase space, based on probability distributions inferred from the spectral fits, in particular the electronic/positronic energy and entropy densities.

In Section 4, we explain the quantization of the classical nonthermal partition function and demonstrate that the classical limit applies to the cosmic-ray electron–positron plasma due to the small fugacity. The formalism developed in Sections 2–4 is based on relativistic dispersion relations and designed to be practically suitable for spectral fits in any energy range. Foundational aspects of relativistic statistical mechanics and thermodynamics such as the arrow of time and entropy are discussed in Refs. [6–9]. Recent applications of relativistic statistical mechanics include the quark–gluon plasma [10,11], plasmas in gravitational fields [12], gases of neutral particles with long-range spin–spin interaction [13] and condensation effects in relativistic Bose–Einstein gases [14]. Kinetic theory with relativistic dispersion relations, in particular the relativistic Boltzmann equation in Grad's moment expansion and Chapman–Enskog approximation, is discussed in Refs. [15,16] and a relativistic version of the Fokker–Planck equation in Ref. [17].

In Section 5, we perform least-squares fits to the AMS-02 and HESS [18] electron and positron spectra. The measured spectra are located in the GeV and low TeV range, where we can use the ultra-relativistic approximation of the spectral densities derived in Sections 2–4, which means to drop the mass term in the electronic dispersion relation. The ultra-relativistic electronic/positronic flux densities obtained in this way are then used to assemble the positron fraction, which has been measured by the AMS-02 Collaboration in the GeV range [19], thus providing a test of the spectral number densities on which the thermodynamic variables are based. Extrapolation of the analytic formula for the positron fraction into the TeV range suggests an extended spectral peak centered around 1 TeV and exponential decay above 10 TeV.

As mentioned, this primordial approach to the cosmic-ray electron–positron plasma is alternative to the use of kinetic equations which require the assumption of specific electron/positron production mechanisms [20]. A possible mechanism is electron/positron emission caused by interaction of high-energy protons or heavier cosmic-ray nuclei with the interstellar medium [3,21]. Quantitative models thereof turn out to be inconsistent with the pronounced rise of the positron fraction, so that additional positron sources have to be invoked, for instance, discrete high-energy sources such as pulsar magnetospheres or the shocked plasmas of supernova remnants [22], even if isotropy is somewhat compromised. Other authors prefer positron production via decay or annihilation of a variety of hypothetical dark matter particles at TeV energies [23] which could exist in galactic halos.

In Section 5, we also give estimates of the specific energy and number densities, entropy production, temperature, fugacity and chemical potential of the electronic/positronic plasma components. We calculate the specific densities of the actually measured electrons/positrons in the energy range above 1 GeV up to low TeV energies where an exponential cutoff occurs. We then restore the electron mass in the dispersion relation and extrapolate the analytic spectral densities obtained from the spectral fits to lower energies. In this way, we can predict the specific energy, number and entropy densities of the complete statistical ensemble comprising all gas particles by integrating the electronic/positronic spectral densities down to the electron mass. In Section 6, we present our conclusions.

2. Relativistic plasma in stationary non-equilibrium

We start with the classical number density of a free electron plasma,

$$\frac{\mathrm{d}\rho_{\mathrm{B}}}{\mathrm{d}E} = \frac{s}{2\pi^2} E^2 \mathrm{e}^{-\beta E - f(E) - \alpha} \sqrt{1 - m^2/E^2}, \tag{2.1}$$

$\hbar = c = 1$, where m is the electron mass, s the spin multiplicity, $\mathrm{e}^{-\alpha}$ the fugacity and $\beta = 1/(k_{\mathrm{B}}T)$ the temperature parameter. $f(E)$ is an empirical spectral function with logarithmic energy dependence, which we parametrize as $f(E) = -\log(g(E)/g(m))$, where $g(E)$ is a positive but otherwise arbitrary function obtained from a spectral fit. The normalization with $g(m)$ means $f(m) = 0$, and the classical Maxwell–Boltzmann (or Jüttner) equilibrium distribution of a free relativistic electron gas is recovered in the limit of vanishing $f(E)$, see Ref. [9] and references therein.

The number density (2.1) is assembled with the integration measure $s\mathrm{d}^3p/(2\pi)^3$ and the dispersion relation for free relativistic particles $p = \sqrt{E^2 - m^2}$, so that $\mathrm{d}^3p = 4\pi\sqrt{1 - m^2/E^2}E^2\mathrm{d}E$. The normalization factor $s/(2\pi)^3$ arises from