Physica A 451 (2016) 171–179

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An analysis of multifractal characteristics of API time series in Nanjing, China

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HIGHLIGHTS

- Multifractal characteristic of API time series is studied based on MF-DFA and singularity spectrum.
- Strength of distribution multifractality is stronger than that of correlation multifractality.
- Temporal variation in structure of API time series is mainly related to long-range correlations.

ARTICLE INFO

Article history:
Received 7 October 2014
Received in revised form 21 November 2015
Available online 1 February 2016

Keywords:
Multifractality
Multifractal detrended fluctuation analysis
Multifractal characteristics
Long-range correlations
A broad probability density function
API

ABSTRACT

This paper describes multifractal characteristics of daily air pollution index (API) records in Nanjing from 2001 to 2012. The entire daily API time series is first divided into 12 parts that serve as research objects, and the generalized Hurst exponent is calculated for each series. And then, the multifractal sources are analyzed and singularity spectra are shown. Next, based on a singularity spectrum, the multifractal-characteristics parameters (maximum exponent α_0, spectrum width $\Delta\alpha$, and asymmetry $\Delta\alpha_{as}$) are introduced. The results show that the fractality of daily API for each year is multifractal. The multifractal sources originate from both a broad probability density function and different long-range correlations with small and large fluctuations. The strength of the distribution multifractality is stronger than that of the correlation multifractality. The variation in the structure of API time series with increasing years is mainly related to long-range correlations. The structure of API time series in some years is richer. These findings can provide a scientific basis for further probing into the complexity of API.

1. Introduction

Air pollution in China is a severe problem with social, economic and political consequences [1]. A World Bank study reported that 16 out of the top 20 most polluted cities in the world were located in China [2]. Air pollution index (API) is a referential parameter frequently used for reporting the levels of ambient air pollution. Three major pollutants, including respirable particulate matter (PM_{10}), sulfur dioxide (SO_2) and nitrogen dioxide (NO_2), have been selected to report daily API in China [3]. Since June 2000, the State Environment Protection Agency of China has required daily API reports, helping the public to understand local air quality. The higher the API, the more serious the air pollution.

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http://dx.doi.org/10.1016/j.physa.2016.01.061

Air pollution system is very complex. The structure of API time series and its temporal variation are rather complicated [4]. The structural complexity leads us to adopt multifractal formalism to analyze its structural characteristics [5]. This formalism is a widely used technique for quantitatively delineating the nonlinear evolution of a complex system and the multiscale characteristics of physical quantity, and aids in an understanding of the intrinsic regularity and the mechanism of physical changes [6]. Multifractal detrended fluctuation analysis (MF-DFA) [7], an important approach in the study of the multifractal properties of a non-stationary time series, has been a favored research tool. Based on MF-DFA, a singular spectrum is obtained, and the important information of multifractal characteristics can be extracted from it. The time-series structural complexity is inferred through this information [7].

MF-DFA and its related methods [8–11] have been widely used in air pollution studies [12,13] and in other relevant fields [14]. For instance, Shi et al. found that APIs from July 2000 to June 2006 in Shanghai were characterized by scale invariance, long-range dependence and multifractal scaling [13]. Diosdado et al. found that time series of air pollutant concentration, such as concentrations of ozone, sulfur dioxide, carbon monoxide, nitrogen dioxide and PM_{10} particles, from 1990 to 2005 in the metropolitan zone of Mexico city, are multifractal [5]. Zhu et al. and Tong et al. investigated the characteristics of air pollutants and the impact of haze in Nanjing, respectively [15,16]. And Shen et al. studied the detrended cross correlation between API and meteorological elements in Nanjing [17,18].

Although the fractal characteristics of API in some Chinese cities and the detrended cross-correlation between API and its influencing factors in Nanjing have been investigated, there is little direct evidence of the yearly fractality, multifractal sources and the variation in fractality with time in Nanjing. Furthermore, the mechanism of variation in fractality has not been fully clarified and developed. The nonlinear mechanism of temporal variation of API in other cities has rarely been reported in the literature [3,19], and it is not clear that the conclusions drawn from other studies are applicable to Nanjing.

In this paper, we analyzed API records for the period 2001–2012 in Nanjing, China, choosing Nanjing because it is the capital city of Jiangsu Province, and an important transportation and industrial center within the province. Moreover, Nanjing's economy has grown rapidly over the past two decades. Expansions in urban population, industrial production, transportation and traffic infrastructure have accelerated the emission levels of various air pollutants.

Because of their role in the prevention and control of atmospheric pollution, local government requirements should be taken into account in API forecasts. However, to date, it is not clear whether the API in Nanjing can be fully predicted. If an API time series is long-range correlated, predictability becomes possible. Our objective is thus to investigate the multifractal characteristics of API in Nanjing in order to find a pattern of long-range correlations. Our main contribution is that our focus is not only on empirical evidence of API multifractality in Nanjing, but also on investigating the multifractal sources and the variations in multifractal strengths with increasing year. Our findings can contribute to an understanding of the structural complexity of API time series.

The structure of this paper is as follows. Section 2 briefly describes methods and data sources. Section 3 provides detailed empirical results. Section 4 provides a discussion of the results, and the last section provides our conclusions.

2. Methodology and data sources

2.1. Methodology

As opposed to simple fractals described by a single scaling exponent, multifractal time series are characterized by a hierarchy of scaling exponents that describe the different scaling behavior of many interwoven subsets of the series. The MF-DFA procedure is briefly described as follows [7].

The original time series in the presence of nonstationarity, $x(k),\ k = 1, 2, \ldots, N$, is integrated to produce the profile $X(i) = \sum_{k=1}^{i}(x(k) - \bar{x})$, $i = 1, 2, \ldots, N$, where $\bar{x} = \sum_{k=1}^{N} x(k)/N$ is the average. Next, the entire time series $X(i)$, $i = 1, 2, \ldots, N$, is divided into $N_s = N - s$ overlapping boxes with window size s [20], each containing $s + 1$ values. The detrended variance $f^2(s, i)$ of the residuals in a box of size s that starts at i and ends at $i+s$ is calculated as in Eq. (1) [20]

$$f^2(s, i) = \left(\sum_{k=i}^{i+s} (X(k) - \tilde{X}(k, i))^2 \right) \Big/ (s + 1) \tag{1}$$

where the local trend, $\tilde{X}(k, i)$, is the ordinate of a least-squares fit with a straight line or higher order polynomial [20]. Here, we adopted "overlapping boxes" [20] instead of "non-overlapping boxes" [21] to calculate detrended variance. Enough boxes can thus be calculated so that the detrended variance becomes stable. Finally, a qth order detrended fluctuation function is calculated as Eq. (2)

$$F_q(s) = \left\{ \frac{1}{Ns} \sum_{i=1}^{Ns} [f(s, i)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \quad \text{for } q \neq 0$$
$$F_0(s) = \exp\left\{ \frac{1}{2Ns} \sum_{i=1}^{Ns} \ln f(s, i) \right\} \quad \text{for } q = 0. \tag{2}$$