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Multifractal Value at Risk model

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HIGHLIGHTS

- Multifractal Value at Risk (MFVaR) is developed in consideration of the multifractal property of financial time series.
- MFVaR is a parametric model and not based on simulation.
- MFVaR can provide a stable and accurate forecasting performance in volatile financial market where large loss can be incurred.

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ABSTRACT

In this paper new Value at Risk (VaR) model is proposed and investigated. We consider the multifractal property of financial time series and develop a multifractal Value at Risk (MFVaR). MFVaR introduced in this paper is analytically tractable and not based on simulation. Empirical study showed that MFVaR can provide the more stable and accurate forecasting performance in volatile financial markets where large loss can be incurred. This implies that our multifractal VaR works well for the risk measurement of extreme credit events.

1. Introduction

Financial time series have various stylized facts and some of which are not sufficiently explained by conventional statistical models. The application of multifractal model to financial time series is a rigorous approach for the analysis of these stylized facts such as heavy tailed distribution, volatility clustering, long memory, and scaling property.

Lux [1], Calvet and Fisher [2], Bacry, et al. [3] and Wei and Wang [4] showed that multifractal approaches have advantages to analyze stylized facts observed in financial market data. Bartolozzi et al. [5], Kumar and Deo [6], Morales et al. [7], Morales et al. [8] and Lee and Chang [9] dealt with time varying multifractal properties in financial time series.

Kantelhardt et al. [10] proposed a multifractal detrended fluctuation analysis (MF-DFA) for the reliable measurement of multifractal scaling behavior of nonstationary time series. Horvatic et al. [11] provided a detrended cross-correlation analysis for measuring the multifractality level in nonstationary time series with periodic trends.

Barunik et al. [12] studied the source of multifractality in financial time series and concluded that the heavy tail distribution is the main cause of multifractality. Grech and Mazur [13], Grech and Pamuła [14], and Caraiani [15] claimed that the better understanding of dynamics of multifractality in financial data makes the more accurate financial crisis forecast. Zunino et al. [16], Zunino et al. [17], Oh et al. [18], and Sensoy [19,20] studied multifractal properties in various financial markets and provided some evidence of the close relationship between multifractality and market efficiency.

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The measurement of financial risk based on multifractal analysis is restricted to the simulation based Value at Risk (VaR) model. Liu and Lux [21] and Calvet and Fisher [22] computed VaR in the US bond market and the exchange market for USD/AUD through simulation based on the Markov switching bivariate multifractal model. Bacry et al. [3] and Batten et al. [23] measured VaR in exchange market using multifractal random walk (MRW) [24] and multifractal model of asset returns (MMAR) [25] respectively. Bogachev and Bunde [26] introduced a historical VaR estimation method considering multifractal property of data.

In this paper, we introduce a novel type of multifractal Value at Risk (MFVaR), which is designed using the MMAR process of Mandelbrot et al. [25] with the binomial multifractal measure. MFVaR is a parametric risk measure and analytically tractable. We computed MFVaR for the Korean and US stock market indices (KOSPI and S&P500) and currency exchange (USD/KRW) daily data. Our MFVaR are compared with other benchmark VaR method such as Gaussian, t-distribution, and GARCH simulation method, and showed more stable and accurate results than those obtained from other conventional methods especially in the volatile foreign exchange market data.

The remaining structure of this paper is as follows. Some theoretical background for MFVaR is reviewed in Section 2. The details of MFVaR model is provided in Section 3, and the application of MFVaR to the real market data and its comparison with other conventional VaR models are provided in Section 4. Section 5 presents summary and conclusion.

2. Theoretical background

In this section, we review the Multifractal properties and the related research since Mandelbrot et al. [25]. Binomial multifractal measure and the time deformation concept are described and how to estimate the model parameters is introduced.

2.1. Multifractal property

When a time series, $X(t)$, satisfy the property in Eq. (1), $X(t)$ is called a self-similar process.

$$\{X(\lambda t)\} \overset{d}{=} \{\lambda^H X(t)\} \tag{1}$$

where H is a scaling exponent or Hurst exponent and controls the long memory property of $X(t)$. When the increment of $X(t)$, $X(t+\tau)-X(t)$, is stationary, H exists between 0 and 1. When H is 0.5, increments are independent. If H is larger than 0.5, $X(t)$ is persistent and have long memory property in returns, while $X(t)$ is antipersistent if H is smaller than 0.5 [27].

The fractal dimension, D, is expressed using H as follows (see Refs. [28–30]).

$$D = 2 - H. \tag{2}$$

While monofractal processes such as self-similar process have a constant fractal dimension, and multifractal processes have multiple fractal dimensions. The following scale property of time series in Eq. (1) distinguishes multifractal processes from monofractal processes.

$$\mathbb{E}[|X(t+\tau) - X(t)|^q] \sim \tau^{\zeta_X(q)} \tag{3}$$

where $\zeta_X(q)$ is a scaling function. When $X(t)$ is multifractal, $\zeta_X(q)$ exhibits a nonlinear (concave) formation, and $\zeta_X(q)$ becomes linear ($\zeta_X(q) = Hq$) when $X(t)$ is monofractal (e.g. self-similar process).

2.2. Review of multifractal time-series models

Let $X(t)$ be the log-price of financial asset. Based on multifractal model of asset returns (MMAR), $X(t)$ is written as follows.

$$X(t) = B_H[\theta(t)] \tag{4}$$

where $B_H(t)$ is a fractional Brownian motion (fBm) and $\theta(t)$ is a time deformation process called trading time. The multifractality of MMAR process stems from the time deformation of $\theta(t)$, which turns a time sequence into a multifractal series, inside of self-similar process, $B_H(t)$. Both $\theta(t)$ and $B_H(t)$ are independent of each other and $\theta(t)$ is expressed as a multifractal measure.

The multifractal measure, $\theta_k(t)$, defined on $t \in [0, 1]$ is constructed through k-step multiplicative cascade.

$$\theta_k(t) = \theta_k[0, t] = \sum_{i=1}^{N} \theta_k[it, it + \Delta t] \tag{5}$$

$$\theta_k[t, t+\Delta t] = M_{\eta_1} M_{\eta_1\eta_2} \cdots M_{\eta_1\cdots\eta_k} \tag{6}$$

where $\Delta t = b^{-k}$, $N = b^k$, $\eta_i \in \{0, 1, \ldots, b-1\}$, and M is a positive, independent, and identical random variable whose value belongs to $S = \{m_0, m_1, \ldots, m_{b-1}\}$. As k goes to infinity, $\theta_k(t)$ converges to $\theta(t)$ in Eq. (4).