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Nonextensive statistical mechanics approach to electron trapping in degenerate plasmas

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HIGHLIGHTS

- The electron trapping in a degenerate plasma is reformulated by incorporating the nonextensive entropy prescription.
- The q-deformed Fermi–Dirac distribution function including the quantum as well as the nonextensive statistical effects is used.
- A new generalized electron density is obtained.
- The modifications arising in the propagation of ion-acoustic solitary waves are analyzed.

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ABSTRACT

The electron trapping in a weakly nondegenerate plasma is reformulated and re-examined by incorporating the nonextensive entropy prescription. Using the q-deformed Fermi–Dirac distribution function including the quantum as well as the nonextensive statistical effects, we derive a new generalized electron density with a new contribution proportional to the electron temperature T, which may dominate the usual thermal correction ($\sim T^2$) at very low temperatures. To make the physics behind the effect of this new contribution more transparent, we analyze the modifications arising in the propagation of ion-acoustic solitary waves. Interestingly, we find that due to the nonextensive correction, our plasma model allows the possibility of existence of quantum ion-acoustic solitons with velocity higher than the Fermi ion-sound velocity. Moreover, as the nonextensive parameter q increases, the critical temperature T_c beyond which coexistence of compressive and rarefactive solitons sets in, is shifted towards higher values.

1. Introduction

Quantum plasmas [1,2] have recently gained growing interest driven mainly by the space observations (neutron stars, white dwarfs etc.) [3–5] and recent laboratory experiments (ultrafast thermalization of laser plasmas [6], free electron laser excited plasmas [7], inertial confinement fusion experiments [8] etc.). On the other hand, quantum plasmas are presently of increasing interest connected mainly with their potential applications in modern technology [9,10] (metallic and semiconductor nanostructures such as metallic nanoparticles, metal clusters, thin metal films, spintronics, nanotubes, quantum wells and quantum dots etc.). The ongoing miniaturization of semiconductor devices and nanoscale objects has made it possible to envisage practical applications of quantum plasmas. The great degree of miniaturization of today's electronic components is such that the de Broglie wavelength of electrons λ_B is comparable with the average interelectron distance r_0 and $\lambda_B \ll \lambda_L$, where λ_L is the Landau length [11], i.e., the distance over which the characteristic kinetic energy

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$k_B T$ (where k_B ant T stand, respectively, for the Boltzmann constant and temperature) of thermal motion equals the potential energy of interaction between charges. There has been a renewed interest in quantum plasmas, covering different plasma modes, instabilities, and other nonlinear effects [12–22]. More recently [22], we proposed to re-examine the quantum ion-acoustic (QIA) waves, by adding the effect of another quantum contribution: the exchange–correlation potential for the electrons [23]. This effect, resulting from the total antisymmetry of the electron wavefunction, is generally not negligible when the quantum effects start playing a role. It has been shown that the exchange–correlation effect may drastically influence the main quantities (phase velocity, amplitude and width), and existence conditions of the QIA solitary waves. Note that this effect attracted a good deal of interest because of the insight it may lend into the formation and dynamics of nonlinear structures [24–26]. Different quantum transport models have been developed [19]. The quantum hydrodynamic (QHD) model consists of a closed set of equations describing the transport of charge, momentum and energy in a charged particle system interacting through a self-consistent electrostatic potential. This model is considered as an extrapolation of the classical fluid model, with a new contribution arising from the so-called Bohm potential due to the quantum effects. Note that the QHD model equations can be obtained from the self-consistent Hartree equations [10] or from the phase-space Wigner–Poisson equations [27]. The QHD model has some well-known limitations [28], and it has been shown that it is valid only in the case of weak coupling [29].

Particle trapping [30–35] is a nonlinear effect, common in space plasmas as well as in laboratory and numerical experiments. Some of the plasma particles are confined (trapped) by the wave to a finite region of the phase space where they bounce forth and back. The first analytical method to construct equilibrium electrostatic structures involving particle trapping was given by Bernstein, Greene and Kruskal (BGK) [36]. Later on, Gurevich [37] considered trapping as a microscopic phenomenon. To overcome the difficulties of the BGK method, Schamel [38] developed a different method of constructing equilibrium solutions, called pseudopotential method. The original work of Schamel and subsequent papers allowed then a breakthrough in the theory of holes or phase space vortices. Recently, Shah et al. [39] investigated the effect of trapping on the formation of solitary structures in a degenerate quantum plasma. They used the Fermi–Dirac distribution function for the electrons with arbitrary degeneracy and obtained an expression for the trapped electrons density in a potential well. Note that relatively little work has been done on trapping as microscopic phenomena in quantum plasmas [39,14,40].

In an earlier paper [41], the Fermi statistical model has been reformulated by incorporating the nonextensive entropy prescription [42]. Note that over the last few years, there has been a renewed interest in nonextensive plasmas, covering different plasma modes, instabilities, and other collective phenomena effects (see Refs. [43–46] and references therein for an actual view of the theory and its breadth of use). This interest has been mainly motivated by the fact that during the last two decades, it has been proven that systems endowed with long-range interactions, long-time memory, fractality of the corresponding space–time/phase-space, or intrinsic inhomogeneity are untractable within the conventional Boltzmann–Gibbs (BG) statistics. To overcome this shortcoming, Tsallis [42], in a celebrated and influential paper, proposed a nonextensive generalization of the BG entropy. This generalization was first recognized by Renyi [47] and subsequently proposed by Tsallis, suitably extending the standard additivity of the entropies to the nonlinear, nonextensive case where one particular parameter, the entropic index q, characterizes the degree of nonextensivity of the system under hand (for $q \to 1$, the standard BG statistics is recovered).

To the best of our knowledge, the combined quantum and nonextensive statistical effects on particle trapping in quantum plasmas have never been considered in the literature. It is therefore a matter of some interest to reformulate and re-examine the electron trapping in a quantum plasma by incorporating the nonextensive entropy prescription. Making use of the generalized Fermi distribution and following Landau and Lifshitz [48], we first derive a new generalized electron density, and than analyze the modifications arising in the propagation of ion-acoustic solitary waves.

2. Theoretical model

a-**Generalized electron density**

We begin by deriving an expression of the generalized number density for degenerate electrons adiabatically trapped in a potential $\phi(r, t)$. Nonextensive statistical mechanics leads to a q-deformed Fermi–Dirac distribution function. The latter can be obtained by a maximization of the q-entropy subject to constraints, namely that the number of particles is constant and that the total energy is constant. This nonextensive generalized Fermi–Dirac distribution including the quantum as well as the nonextensive statistical effects is given by [41]

$$f^{(q)}(E) = \begin{cases} \dfrac{1}{1 + \{1 + (q-1)[(\varepsilon - e\phi - \mu)/T]\}^{q/q-1}} & \text{if } 1 + (q-1)[(\varepsilon - e\phi - \mu)/T] > 0 \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

where $\varepsilon = p^2/2m_e$, p is the electron momentum, m_e is the electron mass, e is the electronic charge, T is the electronic temperature given in energy unit, ϕ is the electrostatic potential, μ is the chemical potential, and $q \geq 1$ stands for the nonextensive parameter. Note that for $q \to 1$, (1) reduces to the well-known Fermi–Dirac distribution function. In the zero-temperature limit, it reduces to the Heaviside step function corresponding to an ideal degenerate Fermi gas, regardless the value of the entropic index q. Following Landau and Lifshitz [48], the electron energy in the presence of a potential field ϕ is