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Optimal control strategy for a novel computer virus propagation model on scale-free networks

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HIGHLIGHTS

- Propose a novel *SLBOS* computer virus propagation model on scale-free networks.
- Obtain the spreading threshold and global stability criterions.
- Investigate existence of an optimal control for the control problem.
- Some numerical simulations are given to illustrate the main results.

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ABSTRACT

This paper aims to study the combined impact of reinstalling system and network topology on the spread of computer viruses over the Internet. Based on scale-free network, this paper proposes a novel computer viruses propagation model—*SLBOS* model. A systematic analysis of this new model shows that the virus-free equilibrium is globally asymptotically stable when its spreading threshold is less than one; nevertheless, it is proved that the viral equilibrium is permanent if the spreading threshold is greater than one. Then, the impacts of different model parameters on spreading threshold are analyzed. Next, an optimally controlled *SLBOS* epidemic model on complex networks is also studied. We prove that there is an optimal control existing for the control problem. Some numerical simulations are finally given to illustrate the main results.

1. Introduction

Computer virus is a kind of computer program that can replicate itself and spread from one computer to others [1]. Computer viruses have caused enormous financial losses in the past few decades. With the popularization of the Internet and wireless networks, the epidemic capability of computer viruses has been overwhelmingly magnified. Indeed, computer viruses have turned out to be a major threat to our work and life [2]. The dynamical modeling, as it is known, is a vital approach to studying the way computer virus spreads on the Internet. Since Kephart and White [3], who followed the idea suggested by Cohen [4] and Murray [5], presented the first propagation model of computer virus, numerous relevant efforts in this field have been done.

In the past decade, the work on computer virus epidemiology has been mainly focused on the following two topics:

(i) Viruses spreading on fully-connected networks that are established based on the assumption that every computer on the network is equally likely to be accessed by any other computer across the network. Some classical models range from

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conventional models, such as the Susceptible–Infected–Susceptible (***SIS***) models [3,6], Susceptible–Infected–Removed (***SIR***) models [7,8], Susceptible–Infectious–Recovered–Susceptible (***SIRS***) models [9–11], Susceptible–Infected–External–Susceptible (***SIES***) models [12], Susceptible–Exposed–Infectious–Recovered (***SEIR***) models [13,14], Susceptible–Exposed–Infectious–Recovered–Susceptible (***SEIRS***) models [15], Susceptible–Exposed–Infectious–Quarantined–Recovered–Susceptible (***SEIQRS***) models [16], Susceptible–Latent–Breaking–Susceptible (***SLBS***) models [17–22], Susceptible–Infected–Countermeasure–Susceptible (***SICS***) models [23,24], and some other models [25–28], to unconventional models such as delayed models [29–36] and stochastic models [22,37].

(ii) Viruses spreading on complex networks, which was stimulated by the discovery that the Internet follows a power-law degree distribution [38–40]. These pioneering work has aroused intense interest in the impact of network topology on virus spreading. As a result, multifarious network-based virus epidemic models, ranging from Susceptible–Infected (***SI***) models [41–43] and ***SIS*** models [44–47] to ***SIR*** models [48–52] and ***SLBS*** models [53] have been inspected.

To better understand the combined impact of both reinstalling system and network topology on virus spreading, in this paper we propose a novel ***SLBOS*** model by assuming that the underlying network is scale-free, i.e. its degree distribution follows a power law distribution. A comprehensive study of the model is conducted. Specifically, the spreading threshold R_0 is calculated, the virus-free equilibrium is shown to be globally asymptotically stable if $R_0 < 1$, and the viral equilibrium is proved to be permanent if $R_0 > 1$. To better control computer virus propagation, an optimally controlled ***SLBOS*** epidemic model on complex networks is also proposed.

The layout of this paper is as follows: Section 2 deals with the relevant mathematical framework (notations, hypotheses, and model formulation). Section 3 determines the spreading threshold and equilibria for this model. In Sections 4 and 5, we examine the global stability of the virus-free equilibrium and the permanence of the viral equilibrium, respectively. The effects of system parameters on virus spreading are analyzed in Section 6. In Section 7, the analysis of optimization problems is presented. Some numerical simulations are performed in Section 8. Finally, Section 9 outlines this work.

2. Description of the new model

We shall use a graph $G = (V, E)$ to represent the Internet topology, where nodes and edges stand for computers and communication links among computers, respectively. Its node degrees asymptotically comply with a power-law distribution, $P(k) \sim k^{-\tau}$, where $P(k)$ means the probability that a node chosen randomly from the Internet is of degree k [39,40]. At any time a node has two states: within system, and without system. The nodes within system have three states: virus-free nodes, infected nodes that are latent, and infected nodes that are breaking out. Hence, the total population of nodes is divided into four groups: uninfected nodes within system (***S***-nodes), latent nodes within system (***L***-nodes), breaking out nodes within system (***B***-nodes) and nodes without system (***O***-nodes).

For convenience, let us introduce some quantities as follows.

- Λ: the maximum node degree of graph G.
- $S_k(t)$: the relative density of k-degree S-nodes.
- $L_k(t)$: the relative density of k-degree L-nodes.
- $B_k(t)$: the relative density of k-degree B-nodes.
- $O_k(t)$: the relative density of k-degree O-nodes.
- $S(t) := (S_1(t), S_2(t), \ldots, S_\Lambda(t))$.
- $L(t) := (L_1(t), L_2(t), \ldots, L_\Lambda(t))$.
- $B(t) := (B_1(t), B_2(t), \ldots, B_\Lambda(t))$.
- $O(t) := (O_1(t), O_2(t), \ldots, O_\Lambda(t))$.

Let us make the following assumptions:

(H_1) All nodes are virus-free when installing the system.
(H_2) Owing to installing system, every O node enters the Internet with probability per unit time $\delta > 0$.
(H_3) Owing to uninstalling system, every node within system leaves the Internet with probability per unit time $\mu > 0$.
(H_4) When communicating with a single neighboring L-node (respectively, B-node), an S-node gets infected and, hence, becomes a L-node with constant probability $\beta_1 > 0$ (respectively, $\beta_2 > 0$).
(H_5) Owing to possible outbreak of viruses, a L-node becomes a B-node with constant probability $\alpha > 0$.
(H_6) Owing to the effort in purging viruses, a B-node gets cured and, hence, becomes an S-node with constant probability $\gamma > 0$.
(H_7) The total number of nodes does not change or, equivalently, $S_k(t) + L_k(t) + B_k(t) + O_k(t) = 1$.
(H_8) The probability that a link has a L-node as one endpoint does not hinge on the degree of the other endpoint of the link and, hence, is only a function of $L(t)$.

Likewise, the probability that a link has a B-node as one endpoint is only a function of $B(t)$. Let $\Theta_1(L(t))$ and $\Theta_2(B(t))$ denote these two probabilities, respectively. Let $\langle k\rangle$ stand for the average node degree, $\langle k\rangle := \sum_k kP(k)$. Direct probabilistic calculations yield

$$\Theta_1(L(t)) = \frac{1}{\langle k\rangle}\sum_k kP(k)L_k(t), \qquad \Theta_2(B(t)) = \frac{1}{\langle k\rangle}\sum_k kP(k)B_k(t). \tag{1}$$