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A method of examining the structure and topological properties of public-transport networks

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HIGHLIGHTS

- Thorough review of fundamental works in graph theory and network science.
- Combining programming, network and statistical data analysis in transport studies.
- Developing a method to examine and analyze complex public-transport networks.
- Applying the method to explore and analyze real-world public-transport network.
- Analyzing the results and outlining directions for method's extension and research.

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ABSTRACT

This work presents a new method of examining the structure of public-transport networks (PTNs) and analyzes their topological properties through a combination of computer programming, statistical data and large-network analyses. In order to automate the extraction, processing and exporting of data, a software program was developed allowing to extract the needed data from General Transit Feed Specification, thus overcoming difficulties occurring in accessing and collecting data. The proposed method was applied to a real-life PTN in Auckland, New Zealand, with the purpose of examining whether it showed characteristics of scale-free networks and exhibited features of "small-world" networks. As a result, new regression equations were derived analytically describing observed, strong, non-linear relationships among the probabilities of randomly chosen stops in the PTN to be serviced by a given number of routes. The established dependence is best fitted by an exponential rather than a power-law function, showing that the PTN examined is neither random nor scale-free, but a mixture of the two. This finding explains the presence of hubs that are not typical of exponential networks and simultaneously not highly connected to the other nodes as is the case with scale-free networks. On the other hand, the observed values of the topological properties of the network show that although it is highly clustered, owing to its representation as a directed graph, it differs slightly from "small-world" networks, which are characterized by strong clustering and a short average path length.

1. Introduction

Recent findings in network science [1,2] have provided researchers with new tools by which to examine, understand and analyze complex network topologies, thus enabling transport planners to discover features characterizing the structure of

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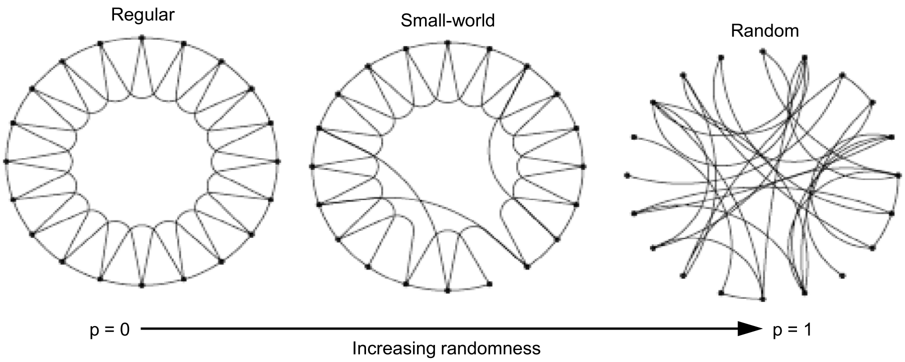

Fig. 1. Random rewiring procedure of the Watts–Strogatz model [7].

public-transport networks (PTNs)—a serious prerequisite towards successfully improving the structure of existing or the design of new PTNs [3].

1.1. Objectives

The lack of a well-defined approach enabling systematic exploration and subsequent analysis of PTNs and dealing with the emerging difficulties and constraints concerning data availability, motivated this study, the main objective of which is to develop an efficient method, coupled with a software tool, of allowing automated extraction and processing of data, and hence of examining and analyzing the structure and topological properties of PTNs.

The purpose of this work is to develop a novel method to bridge between the theory of complex networks and their practical application, for exploring and analyzing real world complex PTNs. The method will be based on comprising tools and knowledge of computer programming and large-network and statistical data analysis.

2. Literature review: examination of complex networks

The past few decades have seen a great interest in empirical studies focusing on the network structure and topological properties of Public Transport (PT), whether by road, rail, sea or air [4]. Complex real-life systems like PTNs have continuously been an object of in-depth studies, marked by a large number of works related to Graph Theory and Network Science. These emerged as a result of the notable work, among others, of Erdős and Rényi [5,6] on the theory and practice of Random Graphs, and subsequently continued with the contributions of Watts and Strogatz [7] on "Small-World" Networks (SWN), Newman [8] on the "small-world" effect, Barabási and Albert's [9] findings in Scale-Free Networks (SFN), Latora and Marchiori's [10] introduction of the term "efficiency" and its application in public transport [11].

In their study, Watts and Strogatz [7] found that in terms of structural properties, networks were highly clustered, similar to the regular lattice, and had small average path lengths (APL) specific to random networks (RN). They named these networks "small-world", as is shown in Fig. 1, by analogy with the so-called "small-world" phenomenon [12], also known as six degrees of separation [13]. Watts and Strogatz's work effectively initiated the myriad studies focusing on examining real-life networks for "small-world" properties.
Thus, for example, as a result of their study of the topology [1,14] of large and diverse networks, Barabási and Albert [9] concluded that "*independent of the system and the identity of its constituents, the probability $P(k)$ that a vertex in the network interacts with k other vertices decays as a power-law, following $P(k) \sim k^{-\gamma}$*," where the exponent γ is called a scaling factor.

Barabási and Albert [9] proved that the existing network models, such as Erdős and Rényi (ER) and Watts and Strogatz (WS), failed in two important features of real-life networks: growth and preferential attachment. In random networks (ER), for example, the number of nodes N is fixed, and each two vertices are connected with a given probability p; the values of this probability, with which any vertex has k edges, are distributed according to a Poisson distribution. On the other hand, according to the WS model, "small-world" networks have N vertices forming a one-dimensional lattice, in which each node is connected to two other vertices – the nearest one and the next nearest – with a probability p, whose value is selected randomly. The important feature in the ER and WS models is that the probability p of a randomly selected vertex being highly connected and having a large value for k decreases following an exponential distribution, which means that highly connected vertices in these networks are in practice absent. In contrast, the finding according to which the probability $P(k)$ of a given vertex having k neighbors follows a power-law distribution; within these networks, therefore, it is more likely that highly connected vertices will occur. These are called scale-free networks and are shown in Fig. 2.

In order to prove that power-law scaling requires both features – growth and preferential attachment – simultaneously, Barabási and Albert [9] built two types of models: (i) Albert's growth model (A), which keeps the property's growth and the attachment to which is not preferential but uniform (i.e., each new vertex connects to the existing vertices with equal probability) and (ii) Barabási's preferential attachment model (B), in which the number of vertices is fixed (i.e.,