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Physica A 385 (2007) 773–780
PHYSICA A
www.elsevier.com/locate/physa

Networking effects on evolutionary snowdrift game in networks with fixed degrees

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Received 2 June 2007; received in revised form 19 July 2007
Available online 27 July 2007

Abstract

We study the effects of spatial structures other than the degree distribution on the extent of the emergence of cooperation in an evolutionary snowdrift game. By swapping the links in three different types of regular lattices with a fixed degree k, we study how the frequency of cooperator f_C changes as the clustering coefficient (CC), which signifies how the nearest neighbors of a vertex are connected, and the sharing coefficient (SC), which signifies how the next-nearest neighbors of a vertex are shared by the nearest neighbors, are varied. For small k, a non-vanishing CC tends to suppress f_C. A non-vanishing SC also leads to a suppressed f_C for the networks studied. As the degree increases, the sensitivity of f_C to the network properties is found to become increasingly weak. The result is discussed within the context of the ranking patterns of average payoffs as k changes. An approximation for f_C, which is based on the idea of a finite fully connected network and gives results in good agreement with numerical results, is derived in the limit of large k.

PACS: 87.23.Kg; 89.75.Hc; 02.50.Le; 87.23.Cc

Keywords: Networking effect; Snowdrift game; Clustering coefficient; Sharing coefficient

1. Introduction

Evolutionary games are an important tool in the study of the emergence and extent of cooperative character in systems consisting of competing entities [1–4]. Physicists, applied mathematicians, mathematical biologists and ecologists all find this emergence behavior fascinating. Typically, individuals or agents of different characters interact with their connected neighbors. Earlier studies usually allow an agent to interact with all others in the system [1,5]. This is referred to as the well-mixed situation. In the language of networks, the agents are fully connected. Recent progress in complex networks has led to investigations on the function and effects of an underlying network on the behavior of evolutionary games [6–8]. In general, the geometrical features of a network are described by many properties, degree distribution, clustering coefficients (CCs), path

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doi:10.1016/j.physa.2007.07.038

lengths, etc. It is, therefore, important to study how the cooperative behavior may be affected by the network properties.

The evolutionary prisoner's dilemma game and evolutionary snowdrift game (ESG) [9] are two popular models in exploring how cooperation results from competing agents [10]. The snowdrift game is a slight modification on the classic prisoner's dilemma game. Due to the difficulty in accurately determining the values of payoffs in real systems [11,12], the snowdrift game is treated as a realistic alternative to the prisoner's dilemma. In the present work, we study how the cooperative frequency in ESG behaves in different networks in which the degree of all the nodes is kept fixed. In this way, we are able to investigate how and which geometrical properties affect the extent of cooperation.

The plan of the paper is as follows. In Section 2, we introduce the ESG. In Section 3, we discuss the types of networks on which the ESG is implemented. We intentionally select networks with a fixed degree and then introduce a cut-and-rewire scheme to change the other geometrical properties. Results on the cooperator frequency as a function of rewiring are presented in Section 4. The competition between the degree and effects of rewiring is discussed. An approximated expression for the cooperator frequency is derived for the case of large degrees. We summarize our results in Section 5.

2. Evolutionary snowdrift game

The snowdrift game without evolution is best described within the following context [1,13,14]. Consider two drivers going home in opposite directions on a road and there is a snowdrift blocking their way. Each driver can choose one of two actions: to shovel the snowdrift (cooperate (C)) or not to do anything (not-to-cooperate or "defect" (D)). If the snowdrift is removed, both drivers can go home and thus get a reward of b. Shovelling is a laborious job of cost c. The payoff to a driver depends on the driver's action and his opponent's action. There are several possibilities. (i) The drivers cooperate, and each gets a net reward of $R = b - c/2$. (ii) The drivers defect and they get stuck. Each driver gets a reward of $P = 0$. (iii) If only one driver takes the action C and shovels the snowdrift, then both drivers get through. The driver taking action C gets a "sucker" payoff of $S = b - c$ and the driver taking the action D gets a payoff of $T = b$. In the snowdrift game, the payoffs are ranked according to $T > R > S > P$. This ordering is slightly different from that in the prisoner's dilemma in which $T > R > P > S$. The payoffs in the snowdrift game can then be represented by a 2×2 matrix with T, R, S, P as matrix elements. The cost-to-reward ratio defines the parameter $r = c/(2b - c)$. It is convenient to use r as a parameter in studying the snowdrift game, i.e., we take $T = 1 + r$, $R = 1$, $S = 1 - r$, and $P = 0$ with $0 < r < 1$. Therefore, the payoffs are represented in matrix form as

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix}, \end{array} \tag{1}$$

where the elements represent the payoffs to the action listed in the left-hand column of the matrix.

In ESG, the characters of the players are allowed to evolve, based on their performance relative to their neighbors. The notion of neighbors implies that the structural properties of the underlying network play an important role in ESG. In particular, the cooperator frequency or the percentage of cooperative character among all players is found to depend on the spatial structure [1,5,13]. Given a connection among the players, the character is evolved according to the following procedure. At each time step, a player i is chosen for possible updating. The player i acquires a payoff per neighbor $\bar{V}_i = V_i/k_i$. Thus, $\bar{V}_i$ depends on the character of i as well as the nearest neighbors' characters. The player i then randomly selects a player j among his k_i neighbors for possible evolution by comparing $\bar{V}_i$ and $\bar{V}_j$. The probability that the player i will take on the character of player j is given by $\omega_{ij} = (\bar{V}_j - \bar{V}_i)/(1 + r)$, if $\omega_{ij} > 0$. If $\omega_{ij} < 0$, the character of player i remains unchanged. In this asynchronous updating process, one time step is the time it takes on average for every player to have undergone a possible evolution. This is similar to one Monte Carlo time step in simulating spin systems using a sequential updating scheme. Evolution will cease when all players take on the C or D character. A homogeneous phase of all C (all D) character results for small (large) values of r, for an initially 50–50% distribution of C and D players in the system [13]. For intermediate values of r, a system evolves to a