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On the theory of rheological properties of magnetic suspensions

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Abstract

The paper deals with a theoretical study of influence of magnetic field on effective viscosity of suspension of non-Brownian magnetizable particles. It is supposed that experimentally observed magnetorheological effects are provided by chain-like aggregates, consisting of the particles. Unlike previous works on this subject, we take into account that the chains cannot be identical and estimate their size distribution. The following power law $(\eta-\eta_0)/\eta_0 \sim Mn^{-\Delta}$, detected in many experiments, is obtained theoretically (η and η_0 are the suspension effective viscosity and the carrier liquid viscosity, respectively, Mn is the so-called Mason number, proportional to the shear rate and inversely proportional to the square of magnetic field). The calculated magnitude of the exponent Δ increases with the applied magnetic field from approximately 0.66 to 0.8–0.9 and slowly increases with the volume concentration φ of the particles. These results are in agreement with known experiments.

Keywords: Magnetic suspensions; Internal structures; Rheology

1. Introduction

Magnetorheological fluids are suspensions of magnetizable particles in a carrier liquid. The rheology of these fluids is very attractive since it can be controlled by external magnetic field. The magnetorheological effects appear due to the fact that the field induces a dipole moment of each particle. The dipolar interaction between the particles leads to formation of various heterogeneous aggregates consisting of the particles—linear chains, bulk dense drops, very dense columns, etc. These aggregates are responsible for strong, several decimal orders of magnitude, increase of the suspension effective viscosity under the action of applied magnetic field parallel to the gradient of the suspension flow. When the chains (columns) overlap the chamber (channel) filled by the suspension, the system behaves like a quasi-elastic solid with mechanical properties determined by the applied field, concentration of the particles and their magnetic characteristics as well. If the shear stress, applied to the system, exceeds a certain critical value, usually called as the yield stress, the aggregates break and the material starts to flow. Under these conditions the magnetorheological suspension behaves like a liquid with viscosity determined by the applied field and the shear rate of the flow. A similar rheological behavior is typical for the so-called inverse ferrofluids which are suspensions of the diamagnetic

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(usually plastic) particles in a ferrofluid—colloidal suspension of subdomain magnetic particles in a carrier liquid. Usually diameter of these non-magnetic particles is about micron, the size of the magnetic particles in ferrofluid is about 10 nm, therefore with respect to the plastic particles the ferrofluid presents a homogeneous magnetizable medium.

There are two approaches for theoretical explanation of the rheological effects in the magnetic suspensions. The first approach uses a model of the linear chain-like aggregates consisting of the suspended particles [1–6]. This model is based on the idea that all chains are identical, their size is determined by the balance between magnetic forces, attracting the particles to each other, and the hydrodynamical viscous forces which destroy the chains. The following relation $\eta/\eta_0 - 1 \sim \varphi Mn^{-\Delta}$, with $\Delta = 1$, between the suspension effective viscosity η, viscosity η_0 of the carrier liquid, particle volume concentration φ, shear rate $\dot{\gamma}$ and the applied field H has been obtained and used in these works ($Mn \sim \dot{\gamma}/H^2$ is so-called Mason number, its explicit definition is given below in Eq. (9)).

Another approach [7,8] is based on a model of dense bulk ellipsoidal aggregates, consisting of enormous number of the particles. Actually these bulk drops (columns) represent nuclei of dense phase which appear due to the bulk condensation of the particles. Because of the demagnetizing effects these nuclei are elongated nearly along the applied field. The deviation of the drop axes from the field is determined by the action of the hydrodynamical shear forces.

It is well known from the general theory of the condensation phase transitions, that during the phase separation of initially metastable system, the nuclei of the new phase amalgamate into one simply connected volume. This state of the separated system corresponds to the minimum of the system free energy. However, in the case of the shear flowing suspension with the elongated domains of new phase, the hydrodynamical shear forces deviate the domain axes from the applied field. For the particles inside the drops this deviation is not advantageous thermodynamically. On the other hand, the drop elongation and, therefore, deviation from the field increases with the drop volume. This effect restricts the drop growth. The maximal (stable) volume of the drop is determined by the balance between the thermodynamical effects which tend to amalgamate the nuclei into one drop, on the one hand, and the demagnetizing effects due to the drop deviation from the field, on the other. In the framework of these considerations, under the assumption that any interactions between the drops are negligible and the angle between the drop axis and the field is small, the similar power relation $\eta/\eta_0 - 1 \sim \varphi Mn^{-\Delta}$, however with the exponent Δ being equal to $\frac{2}{3}$ instead of 1 in Refs. [1–6], has been obtained in Ref. [7].

Another model of the ellipsoidal aggregates has been considered in Ref. [8]. The difference between the models [7,8] lies in different assumptions concerning mechanisms of restriction of the drop growth. Unlike thermodynamical approach of Ref. [7], in the model [8] the maximal size of the drop is determined by the balance between normal, to the drop surface, hydrodynamical forces acting on the particle near the drop poles, on the one hand, and the magnetic forces, which attract the particle to the drop, on the other. The above mentioned power law for the suspension viscosity with the exponent $\Delta = 1$ (like in the chain models [1–6]) has been obtained in that model.

In the models [1–8] the magnitudes 1 and $\frac{2}{3}$ of the exponent Δ are fixed. They depend neither on the field H nor on the particle concentration φ. However, in experiments this exponent has never been equal to $\frac{2}{3}$ or 1—it is always anywhere between these values. Moreover, the experiments [2,3,5,7], carried out with the different polar suspensions, demonstrate that Δ increases with the field from, approximately, $\frac{2}{3}$ to, approximately, 1 and slowly increases with the concentration φ. At first sight, these variations of Δ indicate the transformation of the internal structures in the suspension from the drops to the chains. However, the analysis of Ref. [7] demonstrates that increase of the applied field leads to increase of the drop diameter (however, the drop major axis increases faster than this diameter). At the same time, the direct experiments [10] indicate that in the motionless suspensions the opposite transformations from the chains to the drops take place for increasing field. In the light of these results, the transition from the drops to the chains in the shear flowing suspensions looks rather unbelievable, especially with account of the fact that the increase of the exponent Δ with the particle concentration φ cannot be explained in the framework of the hypothesis of the drop-chain transition. It should be noted that in experiments the Mason number Mn is very often either much more or much less than unity. That is why the relatively small fault in the theoretical determination of the exponent Δ leads to large fault in the prediction of the magnitude of the suspension effective viscosity η. It also seems very