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Kinetic, time irreversible evolution of the unstable $\pi^{\pm}$-meson

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Abstract

In Liouville formalism the expression for density matrix, determining the time evolution of unstable $\pi^{\pm}$-meson in the framework of unified formulation of quantum and kinetic dynamics is defined. The eigenvalues problem is investigated in the framework of Prigogine's principles of description of nonequilibrium processes at microscopic level. The problem was solved on the basis of complex spectral representation. It was shown that the approach leads to Pauli master equation for the weakly interacting system.

Keywords: Kinetic; Irreversible; Decay; Dissipative; Nonequilibrium

1. Introduction

Let me examine the eigenvalues problem for the Hamiltonian $H = H_0 + gV$:

$$H|\psi_\alpha\rangle = \tilde{E}_\alpha|\psi_\alpha\rangle, \tag{1}$$

where H_0 is the free Hamiltonian, V the interaction part and g the coupling constant. In the conventional case Hamiltonian H is a Hermitian operator, $\tilde{E}_\alpha$ is a perturbed energy of the state—a real number. It is known that the usual procedure of Eq. (1) solution on the basis of perturbation method can lead to the appearance of the small denominators $1/(E_\alpha - E_{\alpha'})$, where E_α, $E_{\alpha'}$ are the energies corresponding to the unperturbed situation. Obviously, the divergences can arise at $E_\alpha = E_{\alpha'}$. The problem of the small denominators was determined by Poincare as "the basic problem of dynamics" [1]. According to Poincare's classification the systems for which the situation can be corrected are called the "integrable" systems [2,3]. In the opposite case the systems are defined as the "non-integrable". Poincare proved that in the general case the dynamic systems are "non-integrable" ones. The basic question now is—what can we do to avoid Poincare's divergences in case the system is "non-integrable"?

Prigogine and coworkers (Brussels–Austin group) noted that in the general case the satisfactory solution of this problem is impossible on the basis of the conventional formulation of quantum dynamics. The mechanism of asymmetry processes in time, which made it possible to accomplish a passage from the reversible dynamics to the irreversible time evolution, was developed for the solution of this problem. The authors of the approach

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deny the conventional opinion that the irreversibility appears only at the macroscopic level, while the microscopic level must be described by the laws, reversed in the time. The method of description of the irreversibility at the quantum level proposed by them leads to the kinetic, time irreversible equations and determines the connection of quantum mechanics with kinetic dynamics. The approach allows to solve the problems, which could not be solved in the framework of conventional classical and quantum mechanics, for example, now we can realize the program of Heisenberg—to solve the eigenvalues problem for the Poincare's "non-integrable" systems.

We examine the situation using the simple Friedrichs model (the model is presented closely to the text of works [2,4–7]). Despite the fact that the solution of the problem for the Friedrichs model is known [8] it serves as a good example for the demonstration of the essence of situation. The model describes interaction of two level atom and electromagnetic field. In the Friedrichs model $|1\rangle$ corresponds to the atom in its bare exited level [5], $|k\rangle$ corresponds to the bare field mode with the atom in its ground state. The state $|1\rangle$ is coupled to the state $|k\rangle$

$$\begin{aligned} H &= H_0 + gV \\ &= E_1|1\rangle\langle 1| + \sum_k E_k|k\rangle\langle k| + g\sum_k V_k(|k\rangle\langle 1| + |1\rangle\langle k|), \end{aligned} \tag{2}$$

where

$$|1\rangle\langle 1| + \sum_k |k\rangle\langle k| = 1, \quad \langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}; \tag{3}$$

here $\alpha(\alpha') = 1$ or k. The eigenvalues problem for the Hamiltonian H is formulated as follows:

$$H|\psi_1\rangle = \tilde{E}_1|\psi_1\rangle, \quad H|\psi_k\rangle = E_k|\psi_k\rangle. \tag{4}$$

For the eigenstate $|\psi_1\rangle$ (for small g) perturbation method gives the expression

$$|\psi_1\rangle \approx |1\rangle - \sum_k \frac{gV_k}{E_k - E_1}|k\rangle. \tag{5}$$

If $E_1 > 0$, Poincare's divergences appear at $E_k = E_1$.

In accordance with Brussels–Austin group approach the eigenvalues problem can be solved if the time ordering of the eigenstates will be introduced. This procedure can be realized through the introduction into the denominators imaginary terms: $-i\varepsilon$ for the relaxation processes, which are oriented into the future and $+i\varepsilon$ for the excitation processes, which are oriented into the past. In this case the eigenvalues problem (4) is reduced to the complex eigenvalues problem:

$$H|\varphi_1\rangle = Z_1|\varphi_1\rangle, \quad \langle\widetilde{\varphi}_1|H = \langle\widetilde{\varphi}_1|Z_1, \tag{6}$$

$$H|\varphi_k\rangle = E_k|\varphi_k\rangle, \quad \langle\widetilde{\varphi}_k|H = \langle\widetilde{\varphi}_k|E_k, \tag{7}$$

where we must distinguish right-eigenstates $|\varphi_1\rangle$, $|\varphi_k\rangle$ and left-eigenstates $\langle\widetilde{\varphi}_1|$, $\langle\widetilde{\varphi}_k|$ [2,4,5], Z_1 is a complex: $Z_1 = \bar{E}_1 - i\gamma$, $\bar{E}_1$ is a renormalized energy and γ is a real positive value. This procedure makes it possible to avoid Poincare's divergences and leads to the following expressions for the eigenstates $|\varphi_1\rangle$, $\langle\widetilde{\varphi}_1|$ [4]:

$$|\varphi_1\rangle \approx |1\rangle - \sum_k \frac{gV_k}{(E_k - \bar{E}_1 - z)^+_{-i\gamma}}|k\rangle, \tag{8}$$

$$\langle\widetilde{\varphi}_1| \approx \langle 1| - \sum_k \frac{gV_k}{(E_k - \bar{E}_1 - z)^+_{-i\gamma}}\langle k|. \tag{9}$$

In the expressions (8), (9) the designation $1/(E_k - \bar{E}_1 - z)^+_{-i\gamma}$ has been referred to as "delayed analytic continuation" [4,5] and is defined through the integration with a test function $f(E_k)$

$$\int_0^\infty \mathrm{d}E_k \frac{f(E_k)}{(E_k - \bar{E}_1 - z)^+_{-i\gamma}} \equiv \lim_{z\to -i\gamma}\left(\int_0^\infty \mathrm{d}E_k \frac{f(E_k)}{E_k - \bar{E}_1 - z}\right)_{z\in C^+}, \tag{10}$$