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Hydromagnetic parametric resonance instability of two superposed conducting fluids in porous medium

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Abstract

Rayleigh–Taylor instability of a heavy fluid supported by a lighter one through porous medium, in the presence of a uniform, horizontal and oscillating magnetic field is studied. The fluids are taken as viscous (obeying Darcy's law), uniform, incompressible, and infinitely conducting. The amplitude of the oscillating part of the field is taken to be small compared with its steady part. The dispersion relation is obtained in the form of a third-order differential equation, with time as the independent variable and with periodic coefficients, for the vertical displacement of the surface of separation of the two fluids from its equilibrium position. The oscillatory magnetic field of frequency ω and steady part H_0 has a stabilizing influence on a mode of disturbance which is unstable in a steady magnetic field of strength H_0. It is found that the oscillatory magnetic field and porosity of the porous medium have stabilizing effects, while the medium permeability has a destabilizing influence on the considered system. For a constant value of any of these physical parameters, the system has been found to be unstable (for small wavenumbers) as well as stable afterwards after a definite wavenumber value. The marginal stability case of parametric resonance holds when $M_1 = M_2 = 0$ (and hence $m = 0$), in which the characteristic exponents, and the corresponding solutions for u break down, is also investigated in detail. It is found, to order ϵ, that the effect of an oscillating magnetic field has no stabilizing influence on a disturbance which is marginally stable in the steady magnetic field; while to order ϵ^2, and when the magnetic field oscillates, a resonance between this mode of disturbances and the oscillating field leads to instability when $\rho_2 > \rho_1$. It is found also, in this resonant case, that all the constant or varied physical parameters, mentioned above, have destabilizing influences on the considered system. Finally, the other two resonance points appear in non-porous media (i.e., when $m = \pm \mathrm{i}\omega$ and $m = \pm 2\mathrm{i}\omega$), are disappeared here due to the presence of the porous medium.

Keywords: Hydrodynamic stability; Flows through porous medium; Magnetohydrodynamics

1. Introduction

The Rayleigh–Taylor instability [1,2] of a heavy fluid supported by a lighter one has recently assumed importance in magnetohydrodynamics because of its application to such diverse problems as controlled thermonuclear experiments [3], magnetohydrodynamic power generation [4], and rapid acceleration of a plasma by a magnetic field which has application to space engines [5]. Two models of Rayleigh–Taylor

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instability in the presence of a magnetic field are of special interest. In one, considered by Chandrasekhar [6], a heavier fluid is supported against gravity by a lighter fluid, both fluids being infinitely conducting and permeated by a uniform, steady, horizontal magnetic field. In another model, studied by Kruskal and Schwarzschild [7], an infinitely conducting fluid is supported against gravity by a uniform, steady, horizontal magnetic field in vacuum. The fluid is also permeated by a magnetic field of different strength.

The magnetic field generally has a stabilizing influence on the flow of a conducting fluid. This is specially true of the Rayleigh–Taylor and Kelvin–Helmholtz instability problems [8,9]. It is of some interest to investigate whether an oscillating magnetic field has a greater stabilizing influence on the flow of a conducting fluid than a steady magnetic field. Since the oscillating magnetic field produces macroscopic motion in the conducting fluid, any stabilization by such a field may be said to be dynamic stabilization. As in the hydrodynamic case [10,11], and also in the hydromagnetic case, dynamic stabilization is expected to has a stabilizing influence on unstable modes. However, parametric resonance effects in dynamic stabilization may introduce new instabilities in natural stable modes present in the system with a static field. Instabilities arising from parametric resonance in dynamic stabilization may be avoided by choosing the amplitude of the oscillating field to be less than a threshold value [9], so that instabilities may decay because of dissipative forces in the system, or by oscillating the frequency of the oscillatory field suitably. Drazin [12] has investigated the instability of a vortex sheet in a conducting fluid in the presence of an oscillating magnetic field parallel to the flow direction and has come to the conclusion that an oscillating magnetic field is less efficient in stabilizing the vortex sheet than a steady field.

In view of the importance of hydromagnetic Rayleigh–Taylor instability in varied applications, it is of some interest to examine the effect of an oscillating magnetic field on the Rayleigh–Taylor instability problem. Berkowitz et al. [13] made one of the earliest studies dealing with magnetohydrodynamic stabilization. They studied the Rayleigh–Taylor instability of a plasma supported against gravity by a magnetic field of constant magnitude and rotating with a constant angular velocity. When this angular velocity vanishes, the problem is reduced to a special case of the Kruskal and Schwarzschild model of hydromagnetic Rayleigh–Taylor instability in which the fluid is taken to be free from any magnetic field. Berkowitz et al. [13] found that instability survived, however, with a reduced growth rate compared with the case with a static magnetic field studied by Kruskal and Schwarzschild [7]. For recent developments of Rayleigh–Taylor and Kelvin–Helmholtz instabilities, see Refs. [14–18].

Mathieu equation has many applications, e.g., in the phenomenon of parametric resonance which arise in many branches of physics and engineering. One of the important problems is that of dynamic instability which is the response of mechanical and elastic systems to time-varying loads, especially periodic loads. There are many cases in which the introduction of a small vibrating loading can stabilize a system which is statically unstable or destabilize a system that is statically stable. The treatment of the parametric excitation system having many degrees of freedom and distinct natural frequencies is usually operated by using the multiple time scales method as given by Nayfeh [19]. The behavior of such systems is described by an equation of Hill or Mathieu types [20–22]. The linear theory predicts that the amplitude–frequency space of the excitation is divided into regions of growing and decaying solutions. Moreover, the linear model may predict a parametric stability (i.e., decaying response).

On the other hand, flows through porous media has been a subject of great interest for the last several decades. This interest was motivated by numerous engineering applications in various disciplines, such as geophysical thermal and insulation engineering, the modelling of packed sphere beds, the cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, ceramic processes, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, ground water pollution and filtration processes, to name just a few of these applications [23–32]. Much of the recent work on this topic is reviewed by Nield and Bejan [33], Ingham and Pop [34], Vafai [35], and Pop and Ingham [36]. In most previous studies on porous media, treatments based on Darcy's law have been considered. However, it is well known that Darcy's law is an empirical formula relating the pressure gradient, the bulk viscous resistance and the gravitational force in a porous medium. In this case, the usual viscous term in the equation of motion is replaced by the resistive term $-(\mu/k_1)\mathbf{v}$, where μ is the fluid viscosity, k_1 is the medium permeability, and $\mathbf{v}$ is the Darcian (filter) velocity of the fluid. For an excellent work about magnetohydrodynamic flow in porous medium, see Ref. [37].