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The effect of weight on community structure of networks

Ying Fan[a], Menghui Li[a], Peng Zhang[a], Jinshan Wu[b,*], Zengru Di[a]

[a]*Department of Systems Science, School of Management, Beijing Normal University, Beijing 100875, PR China*
[b]*Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1*

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Abstract

The effect of weight on community structures is investigated in this paper. We use weighted modularity Q^w to evaluate the partitions and weighted extremal optimization algorithm to detect communities. Starting from empirical and idealized weighted networks, the matching between weights and edges are disturbed. Then using similarity function S to measure the difference between community structures, it is found that the redistribution of weights does strongly affect the community structure especially in dense networks. This indicates that the community structure in networks is a suitable property to reflect the role of weight.

Keywords: Weighted network; Community structure; Role of link weight

1. Introduction

Link weights, as strength of the interaction represented by networks, are believed to be an important variable in networks. It gives more information about networks besides its topology properties dominated by links. Recently more and more study in complex networks focus on the weighted networks. The problems involve the definition of weight and other quantities which characterize the weighted networks [1–3], the empirical studies of its statistical properties [4–7], evolving models [8–13], and transportation or other dynamics on weighted networks [14–17].

However, how important is the weight, or what significant changes on network structures are induced when weight is changed? This question is related with the role of weight. It should be a fundamental question in the study of weighted networks. But it has not been investigated deeply in the previous studies.

The role of weight should be first investigated by analyzing the correlation between link weight and other properties. In this way, it attempt to answer the question that whether there is some internal mechanism strongly determining weights or not. For example, one may image that link betweenness affects link weight largely because the larger link betweenness implies that the link has more important role in communication on networks, so that the weight on the edge might be also larger. If this is true, the weight should be less

*Corresponding author.
E-mail address: jinshanw@phas.ubc.ca (J. Wu).

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important for the networks because the weight is dependent on links or the weight is determined by the network topology.

In some networks [18,19], the average link weight is proportional to the product of end-point degrees as $\langle w_{ij}\rangle \sim (k_i k_j)^{\theta}$. For instance, link weight depend on the end-point degrees in *E. coli* metabolic network, where the link weights represent the optimal metabolic fluxes [18], and in World Airport Network, where the link weights represent the number of available seats on any given connection for the year 2002 [19]. There are actual flux on these two networks, node degrees could affect link weights by the flux. Here we have investigated such correlations in some social networks. The results are different from metabolic network and World Airport Network. For example, the link weights and the vertex degrees of the scientist collaboration network lack correlations [19].

For the database of our scientific collaboration networks of Econophysicists (EP-SCN) [3], BNU-Email network [20], and Rhesus monkey societies [2,21], based on the standard method for linear correlation analysis in mathematical statistics, we get the correlation results for the link weights and link betweenness. The results are shown in Table 1. All the coefficients of correlation for links are less than 0.25. These negative results reveal that the weight is really an independent variable for social networks. By the way, we also get the correlation coefficients for the vertex weight (strength of vertex) and its degree. They are 0.79, 0.44 and 0.71 for above three networks. The results are rational because both strength and degree are quantities over vertex and related with the number of edges connected onto. In addition, we investigate the correlation between link weight w_{ij} and product of the degrees of the end-point nodes k_i and k_j. As shown in Fig. 1, we found no visible correlation between link weight w_{ij} and the end-point degree $k_i k_j$.

From the above negative conclusion on correlation analysis, we know that link weight is an independent variable at some level. This makes the work on the role of weight more attractive: since it is somehow independent, then how significant is it?

We are going to discuss its significance by considering the difference of network quantities when edge weights are disturbed. First, we can get rid of weights to get an binary corresponding network and compare it

Table 1
Correlation coefficients for weight and other quantities

		EP-SCN	BNU-Email	Monkey
Links	Weight-betweenness	0.0055	0.028	0.19
Vertices	Strength-degree	0.79	0.44	0.71

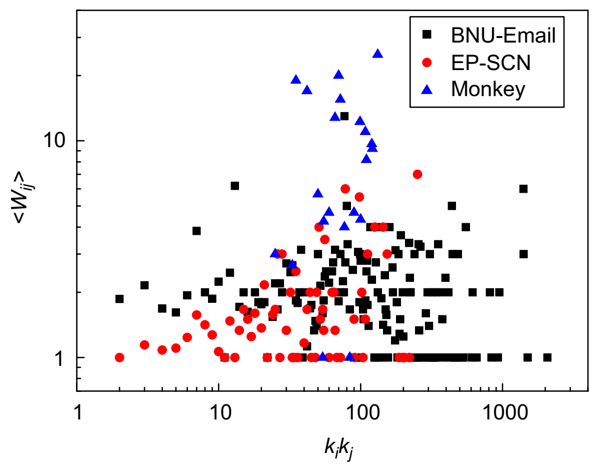

Fig. 1. Scatter plot of link weight and the product of end-point degree. There is no visible correlation and linear analysis between $\ln(w_{ij})$ and $\ln(k_i k_j)$ gives us correlation coefficient 0.0.