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The Ising Spin Glass in dimension four

P.H. Lundow [a,*], I.A. Campbell [b]

[a] Department of Mathematics and Mathematical Statistics, Umeå University, SE-901 87, Sweden
[b] Laboratoire Charles Coulomb (L2C), UMR 5221 CNRS-Université de Montpellier, Montpellier, F-France

HIGHLIGHTS

- We present numerical simulations on Ising spin glass models in dimension 4.
- Finite size and thermodynamic limit regime measurements are analyzed.
- The bimodal and Gaussian interaction model critical exponents are different.
- We conclude that these two models are not in the same universality class.

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ABSTRACT

The critical behaviors of the bimodal and Gaussian Ising spin glass (ISG) models in dimension four are studied through extensive numerical simulations, and from an analysis of high temperature series expansion (HTSE) data of Klein et al. (1991). The simulations include standard finite size scaling measurements, thermodynamic limit regime measurements, and analyses which provide estimates of critical exponents without any consideration of the critical temperature. The higher order HTSE series for the bimodal model provide accurate estimates of the critical temperature and critical exponents. These estimates are independent of and fully consistent with the simulation values. Comparisons between ISG models in dimension four show that the critical exponents and the critical constants for dimensionless observables depend on the form of the interaction distribution of the model.

1. Introduction

Renormalization Group Theory (RGT) for thermodynamic phase transitions [1] and the Edwards–Anderson model for Ising Spin Glasses (ISGs) [2] were introduced almost simultaneously forty years ago. Ever since it has been tacitly assumed as self-evident that the standard RGT universality rules should apply to ISGs. As far as we know there is no rigorous theoretical proof that this ISG hypothesis holds, though confirmations have been reported a number of times based on numerical data [3–6]. The universality principle states that for all systems within a universality class the critical exponents are strictly identical and do not depend on the microscopic parameters of the model. All ISG models in a given dimension are supposed to be in the same universality class, on the assumption that the form of the interaction distribution is an irrelevant microscopic parameter.

Thus in the family of simple ferromagnets, within a universality class of models having space dimension d and spin dimensionality n, all models have identical critical properties corresponding to an isolated fixed point in the renormalization

* Corresponding author.
E-mail address: phl@kth.se (P.H. Lundow).

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group flow. However, diluted ferromagnets of given d, n have a different (dilution independent) set of critical exponents, with values which correspond to a separate isolated fixed point [7,8]. For a few special cases of spin models in dimension two (discussed for instance in Ref. [9]) the critical behavior is more complicated and corresponds to a line of fixed points rather than an isolated fixed point; critical parameters vary continuously according to motion along the line, produced by a marginal operator. From the empirical ISG data below there appear for the moment to be two possible scenarios: two classes of ISGs (such as models with continuous distributions and those with discrete distributions) or alternatively ISG exponents which vary continuously with a parameter such as the kurtosis of the interaction distribution. In any case it has been stated by authoritative authors that "classical tools of RGT analysis are not suitable for spin glasses" [10–12] although no explicit theoretical predictions have been made so far concerning the important question of universality.

Here we combine numerical simulation and high temperature series expansion (HTSE) data on the bimodal and Gaussian ISGs in dimension four so as to obtain accurate and reliable values for the critical parameters in this model. We discuss a number of different methods for exploiting numerical data, and show that for each model these are consistent. Comparisons between these and other estimates on ISG models in the same dimension but with different interaction distributions show that the critical exponents and the critical constants for dimensionless parameters depend on the form of the interaction distribution.

2. Numerical techniques

The Hamiltonian is as usual

$$\mathcal{H} = -\sum_{ij} J_{ij} S_i S_j \tag{1}$$

with the near neighbor symmetric distributions normalized to $\langle J_{ij}^2\rangle = 1$. The Ising spins live on simple hyper-cubic lattices with periodic boundary conditions. We have studied the bimodal model with a $\pm J$ interaction distribution and the Gaussian interaction distribution model. We will compare with published measurements on these and other 4d ISGs. We will use the inverse temperature $\beta = \langle J_{ij}^2\rangle^{1/2}/T = 1/T$, with the normalization above, or alternatively $w = \tanh^2(\beta)$, to signify the temperature. The spin overlap parameter is defined by

$$q = \frac{1}{N}\left\langle \sum_i S_i^A S_i^B \right\rangle \tag{2}$$

where A and B indicate two copies of the same system and N is the number of sites.

The simulations were carried out using the exchange Monte Carlo method for equilibration using so called multi-spin coding, on 2^{14} (up to $L = 7$) or 2^{13} (for larger L) individual samples at each size. An exchange was attempted after every sweep with a success rate of at least 30%. At least 40 temperatures were used forming a geometric progression reaching down to $\beta_{\max} = 0.55$ for the bimodal model and $\beta_{\max} = 0.60$ for the Gaussian model. This ensures that our data span the critical temperature region which is essential for the FSS fits. Near the critical temperature the β step length was at most 0.03. The various systems were deemed to have reached equilibrium when the sample average susceptibility for the lowest temperature showed no trend between runs. For example, for $L = 12$ this means about 200000 sweep-exchange steps.

After equilibration, at least 200000 measurements were made for each sample for all sizes, taking place after every sweep-exchange step. Data were registered for the energy $E(\beta, L)$, the correlation length $\xi(\beta, L)$, and for the spin overlap moments. In addition the correlations $\langle E(\beta, L), U(\beta, L)\rangle$ between the energy and observables $U(\beta, L)$ were also registered so that thermodynamic derivatives could be evaluated using the relation $\partial U(\beta, L)/\partial\beta = \langle U(\beta, L)E(\beta, L)\rangle - \langle U(\beta, L)\rangle\langle E(\beta, L)\rangle$ where $E(\beta, L)$ is the energy [13]. Bootstrap analyses of the errors in the derivatives as well as in the observables $U(\beta, L)$ themselves were carried out.

3. Finite size scaling

ISG simulations are much more demanding numerically than are those on, say, pure ferromagnet transitions with no interaction disorder. The traditional approach to criticality in ISGs has been to study the temperature and size dependence of dimensionless observables, principally the Binder cumulant $g(\beta, L)$ and the correlation length ratio $\xi(\beta, L)/L$, in the near-transition region and to estimate the critical temperature and exponents through finite size scaling (FSS) relations after taking means over large numbers of samples. Finite size corrections to scaling must be allowed for explicitly which can be delicate as the range of sizes L is generally small. On this FSS approach the estimated values for the critical exponents are very sensitive to the critical inverse temperature β_c estimates. Here we first obtain estimates for β_c and the critical parameters using FSS.

The data for standard dimensionless observables, the Binder cumulant

$$g(\beta, L) = \frac{1}{2}\left(3 - \frac{[\langle q^4\rangle]}{[\langle q^2\rangle]^2}\right) \tag{3}$$