Physica A 434 (2015) 99–110

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Cluster synchronization of complex networks via event-triggered strategy under stochastic sampling

Aihua Hu [a,*], Jinde Cao [b,c], Manfeng Hu [a], Liuxiao Guo [a]

[a] School of Science, Jiangnan University, Wuxi 214122, China

[b] Department of Mathematics, Southeast University, Nanjing 210096, China

[c] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia

HIGHLIGHTS

- Mean square cluster synchronization in directed networks is studied.
- The interacting nodes communicate with each other at stochastic periods.
- An event-triggered scheme is proposed based on the sampled-data.
- Virtual leader is supposed to apply pinning control to following nodes.

ARTICLE INFO

Article history:
Received 10 January 2015
Received in revised form 14 March 2015
Available online 18 April 2015

Keywords:
Cluster synchronization
Complex network
Event-triggered scheme
Pinning control
Stochastic sampling

ABSTRACT

This paper is concerned with the issue of mean square cluster synchronization of non-identical nodes connected by a directed network. Suppose that the nodes possess nonlinear dynamics and split into several clusters, then an event-triggered control scheme is proposed for synchronization based on the information from stochastic sampling. Meanwhile, an equilibrium is considered to be the synchronization state or the virtual leader for each cluster, which can apply pinning control to the following nodes. Assume that a spanning tree exists in the subgraph consisting of the nodes belonging to the same cluster and the corresponding virtual leader, and the instants for updating controllers are determined by the given event-triggered strategy, then some sufficient conditions for cluster synchronization are presented according to the Lyapunov stability theory and linear matrix inequality technique. Finally, a specific numerical example is shown to demonstrate the effectiveness of the theoretical results.

1. Introduction

Complex networks widely exist in the real world, such as the Internet, the World Wide Web, biological networks, and multi-agent systems [1–4]. Meanwhile, the well-known network models including small-world and scale-free complex networks have been proposed [5,6]. Complex dynamical networks can display many interesting phenomena, e.g., synchronization. As an important collective behavior, synchronization of complex networks has been focused on in various fields, and many corresponding profound results have been established [7–10].

Cluster synchronization, which can be regarded as a generalization of global synchronization, is considered to be significant in communication engineering and biological science [11,12], but is more challenging. Generally speaking, if

* Corresponding author. Tel.: +86 51085910656.
E-mail address: aihuahu@126.com (A. Hu).

http://dx.doi.org/10.1016/j.physa.2015.03.065

all the nodes in a network split into several clusters (or subgroups), and the nodes in the same cluster rather than different clusters can achieve complete synchronization, then the network is supposed to be in cluster synchronization. At present, some practical control approaches have been proposed for cluster synchronization, such as adaptive control, pinning control, distributed control, and so on [13–20]. However, most of the existing schemes depend on continuous communicating and control updating. Accordingly, the phenomenon of congestion may be caused and the network resources will be wasted. On the other hand, although impulsive control can be utilized to synchronize complex networks [21], which is mainly based on discrete control impulses, the states of the controlled systems are non-continuous. Recently, a new effective strategy being named as event-triggered control [22,23] has been presented. In an event-triggered framework, controllers can be updated discretely, and keep constant by a zero-order-hold device if there is no event being triggered, which implies that the energy can be saved compared with the continuous updating. Furthermore, the event-triggered control is different from the traditional time-triggered control [24], since the main characteristic of the former scheme is that the instants for updating controllers are determined by the given event-triggered condition rather than timing equipment. Consequently, if there is little fluctuation of the controller between two successive instants, then unnecessary updating can be avoided in the event-triggered strategy. There have been a number of works concerning the event-triggered control. For instances, Dimarogonas and Frazzoli et al. [22] proposed centralized and distributed event-triggered schemes for consensus of multi-agents in undirected networks, and the corresponding results were extended to directed networks in Ref. [23]. Besides, self-triggered algorithms were put forward similar to the event-triggered idea [25,26]. In addition, adaptive event-triggered schemes have been investigated [27], and random factors such as noises and packet losses have been considered for network models via the event-triggered control [28,29]. Up to now, to the best of our knowledge, there is few (if any) results concerning cluster synchronization via the event-triggered scheme.

Since a controller in the event-triggered scheme is updated discretely, the continuous system under control will become a switching system. Accordingly, the Zeno behavior maybe exist in the system, which can cause the chattering phenomenon. Fortunately, some improved event-triggered schemes have been demonstrated, such as in Refs. [30,31], event-triggered conditions were supposed to depend on information from interacting nodes just at sampling instants, then event-triggered intervals are obviously greater than zero. Therefore, the Zeno behavior is avoided. However, due to random factors existing everywhere, compared with the fixed period, it is more realistic to introduce stochastic sampling instants [32,33]. Consequently, an event-triggered strategy with stochastic sampling can further extend the current results.

Motivated by the above discussion, this paper focuses on cluster synchronization via an improved event-triggered algorithm, which mainly depends on sampled-data from interacting nodes in the network. Moreover, in order to reflect the reality more closely, the sampling period is supposed to be stochastic. Given that a complex network consists of non-identical nodes with nonlinear dynamics, which split into several clusters and will be pinned by virtual leaders, and the interacting nodes in the network can transmit information with a stochastic period. Then, according to the sampled-data, an event-triggered condition for determining the instants of control updates is proposed, and some sufficient conditions for cluster synchronization will be proved by utilizing the Lyapunov stability theory and linear matrix inequality (LMI) technique. The rest of the paper is organized as follows. Section 2 gives the network model and some preliminaries. Event-triggered condition and sufficient conditions for mean square cluster synchronization via the proposed strategy are shown in Section 3. In Section 4, a numerical example is demonstrated to illustrate the analysis. Finally, conclusions and discussion are provided.

Notation: Let $\mathbb{R}^n$ be the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ be the set of $m \times n$ real matrices. The superscript "T" represents matrix transposition, and the sign $*$ denotes the symmetric block in a symmetric matrix. diag$\{\ldots\}$ represents a diagonal matrix. $\boldsymbol{I}$ denotes the identity matrix. $\mathbf{1}$ and $\mathbf{0}$ denote column vectors with all ones and all zeros, respectively. The norm of a vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$. $\boldsymbol{Y} > 0$ ($\boldsymbol{Y} < 0$) means the symmetric matrix $\boldsymbol{Y}$ is positive (negative) definite. $\mathbb{E}(\cdot)$ indicates the mathematical expectation of a random variable. $\otimes$ represents the Kronecker product.

2. Preliminaries and model formulation

The graph theory [34] is firstly presented, which is useful for later discussion. The topology of a network consisting of N nodes can be described as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \boldsymbol{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the set of nodes, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. If there is a directed edge from node v_j to vertex v_i, which can be denoted by the ordered pair of vertices (v_j, v_i), then the edge $(v_j, v_i) \in \mathcal{E}$, and v_j, v_i are called the parent and child vertices, respectively. The adjacency matrix $\boldsymbol{A} = (a_{ij}) \in \mathbb{R}^{N\times N}$ is defined as follows: When $i \neq j$, if $(v_j, v_i) \in \mathcal{E}$, then $a_{ij} \neq 0$; otherwise $a_{ij} = 0$. In addition, $a_{ii} = 0$ for $i = 1, 2, \ldots, N$. For a node v_i, the set of neighbors can be denoted by $\mathcal{N}(i) = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. Moreover, a path from node v_{i1} to node v_{ik} is a sequence of ordered edges in the form $(v_{i1}, v_{i2}), (v_{i2}, v_{i3}), \ldots, (v_{i(k-1)}, v_{ik})$. The Laplacian matrix $\boldsymbol{L} = (l_{ij}) \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined by $l_{ii} = \sum_{j=1,j\neq i}^{N} a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

In general, a complex dynamical network can be described as:

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{f}_i(\boldsymbol{x}_i(t), t) + \sum_{j\in\mathcal{N}(i)} a_{ij}[\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)], \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\boldsymbol{x}_i(t) \in \mathbb{R}^n$ is the state of the ith node, and $\boldsymbol{f}_i(\boldsymbol{x}_i(t), t) \in \mathbb{R}^n$ represents the nonlinear dynamics of the ith node. $\mathcal{N}(i)$ denotes the set of neighbors for the ith node, and $a_{ij} \neq 0$, which will be discussed later. $\sum_{j\in\mathcal{N}(i)} a_{ij}[\boldsymbol{x}_j(t) - \boldsymbol{x}_i(t)]$ can be taken as the coupling term or the control input.