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Quantum walk, entanglement and thermodynamic laws

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HIGHLIGHTS

- Quantum entanglement between position and chirality and thermodynamic laws.
- We propose a temperature function in order to characterize this equilibrium.
- We show the first and second laws of thermodynamics to the quantum walk (QW).
- We show that the QW entropy has upper and lower bounds.

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ABSTRACT

We consider a special dynamics of a quantum walk (QW) on a line. The walker, initially localized at the origin of the line with arbitrary chirality, evolves to an asymptotic stationary state. In this stationary state a measurement is performed and the state resulting from this measurement is used to start a second QW evolution to achieve a second asymptotic stationary state. In previous works, we developed the thermodynamics associated with the entanglement between the coin and position degrees of freedom in the QW. Here we study the application of the first and second laws of thermodynamics to the process between the two stationary states mentioned above. We show that: (i) the entropy change has upper and lower bounds that are obtained analytically as functions of the initial conditions. (ii) the energy change is associated to a heat-transfer process.

1. Introduction

Quantum walks (QWs) constitute the quantum analogue of classical random walks [1] and also the quantum version of cellular automata [2]. They have been intensively investigated, specially in connection with quantum information science [3–9]. As in the classical case, QWs have been proposed as elements to design quantum algorithms [10–13] and more recently it has been shown that they can be used as a universal model for quantum computation [14,15].

We have been investigating [16–18] the asymptotic behavior of the QW on a line, focusing on the probability distribution of chirality independently of position. We showed that this distribution has a stationary long-time limit that depends on the initial conditions and that it is possible to define a thermodynamic equilibrium between the degrees of freedom of position and chirality [19–22]. For this equilibrium state we have introduced a temperature concept for an unitary closed system.

On the other hand the fundamental lower bound of the thermodynamic energy cost of information processing has been a topic of active research [23,24]. On average the minimum amount of work required to erase 1 bit of information from a memory is $\kappa T \ln 2$ [25]. In the last decades developments in nano-science have enabled the direct measurement of such minuscule amounts of work for small non-equilibrium thermodynamic systems [26]. At the same time recent advances in technology have opened the possibility of building useful quantum computing devices [27]. Therefore, it seems essential to identify bounds on the thermodynamic energy cost of information processing [28] for these new quantum devices.

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In the present paper we study the relationship between the QW thermodynamics and information processing. In particular, we show that it is possible to apply the thermodynamic laws to the QW dynamics after a measurement process. We obtain the upper and lower bound for the asymptotic change of the entanglement entropy. Our result may be thought as complementary to the results presented in Refs. [23,24] where the information content and thermodynamic variables are treated on an equal footing.

The paper is organized as follows. In the next section the usual QW on a line is presented, in the Section 3 the system dynamics with measurement is developed, in the Section 4 the entropy change between the asymptotic stationary states is studied, in the Section 5 the laws of thermodynamics are applied to the same process. Finally in the last section we draw some conclusions.

2. QW on a line

The composite Hilbert space of the QW is the tensor product $\mathcal{H}_T \otimes \mathcal{H}_\pm$ where $\mathcal{H}_T$ is the Hilbert space associated to the motion on a line and $\mathcal{H}_\pm$ is the chirality (or coin) Hilbert space. In this composite space the walker moves, at discrete time steps $t \in \mathbb{N}$, along a one-dimensional lattice of sites $k \in \mathbb{Z}$. The direction of motion depends on the chirality states, either right or left. The wave vector can be expressed as the spinor

$$|\Psi(t)\rangle = \sum_{k=-\infty}^{\infty} \begin{pmatrix} a_k(t) \\ b_k(t) \end{pmatrix} |k\rangle, \tag{1}$$

where the upper (lower) component is associated to the left (right) chirality. The QW is ruled by a unitary map whose standard form is [29–32]

$$\begin{aligned} a_k(t+1) &= a_{k+1}(t)\, \cos\theta \, + b_{k+1}(t)\, \sin\theta, \\ b_k(t+1) &= a_{k-1}(t)\, \sin\theta \, - b_{k-1}(t)\, \cos\theta, \end{aligned} \tag{2}$$

where $\theta \in [0, \pi/2]$ is a parameter defining the bias of the coin toss. Here we take $\theta = \frac{\pi}{4}$ for an unbiased or Hadamard coin. The probability of finding the walker at (k, t) is

$$P(k,t) = |a_k(t)|^2 + |b_k(t)|^2 . \tag{3}$$

The global left and right chirality probabilities are defined as

$$\begin{aligned} P_L(t) &\equiv \sum_{k=-\infty}^{\infty} |a_k(t)|^2 , \\ P_R(t) &\equiv \sum_{k=-\infty}^{\infty} |b_k(t)|^2 , \end{aligned} \tag{4}$$

with $P_R(t) + P_L(t) = 1$ and the interference term is defined as

$$Q(t) \equiv \sum_{k=-\infty}^{\infty} a_k(t) b_k^*(t). \tag{5}$$

In the generic case $Q(t)$ together with $P_L(t)$ and $P_R(t)$ are time depend functions that have long-time limiting values [16] which are determined both by the initial conditions and by the map in Eq. (2). The relation between the initial conditions and the asymptotic distributions has also been recently explored in Ref. [33]. Let us call the mentioned limits as

$$\begin{aligned} \Pi_L &\equiv \lim_{t\to\infty} P_L(t), \\ \Pi_R &\equiv \lim_{t\to\infty} P_R(t), \\ Q_0 &\equiv \lim_{t\to\infty} Q(t) = \mu + \mathrm{i}\nu, \end{aligned} \tag{6}$$

where μ and ν are respectively the real and imaginary part of Q_0. The following relations are verified [16] between Π_L, Π_R and Q_0

$$\begin{aligned} \Pi_L &\equiv \frac{1}{2} + \mu, \\ \Pi_R &\equiv \frac{1}{2} - \mu. \end{aligned} \tag{7}$$

It is important to emphasize that the asymptotic behavior in Eq. (6) is determined by the interference term Q_0 that only depends on the initial conditions.