Physica A 425 (2015) 27–33

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Impact of the traffic interruption probability of optimal current on traffic congestion in lattice model

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HIGHLIGHTS

- A new lattice model is proposed by incorporating the traffic interruption probability of optimal current.
- The interruption probability can efficiently suppress traffic jams under high response coefficient.
- The mKdV equation is affected by the traffic interruption probability.
- The traffic interruption factor can improve the stability of traffic flow with more attention to traffic interruption incident.

ARTICLE INFO

Article history:
Received 11 December 2014
Received in revised form 11 January 2015
Available online 22 January 2015

Keywords:
Traffic flow
Lattice model
Optimal current difference
Interruption probability

ABSTRACT

In this paper, a new lattice model is proposed with the consideration of the traffic interruption probability of the optimal current. The linear stability condition is obtained by linear stability analysis and the mKdV equation is deducted from nonlinear analysis via considering the traffic interruption probability of the optimal current, respectively. The results of numerical simulation show that the traffic interruption probability of the optimal current can efficiently suppress traffic jams under high response coefficient and deteriorate traffic situations under low response coefficient.

1. Introduction

Traffic congestion plays more and more effect on people's lives. To investigate the trait of traffic flow, some new traffic models were proposed recently by considering different traffic conditions [1–11]. In particular, Perc et al. [12,13] and Szolnoki et al. [14] have investigated the subject of lattice modeling of social phenomena by introducing coevolutionary games rules on the evolution of cooperation, which might affect the interaction network. Some researchers [15,16] also revealed interdependent networks to study the effect of interaction between lattice cells to be considered in traffic networks, thereby particularly the dynamical effects of coevolutionary rules in networks traffic will promise exciting new discoveries. In real traffic, various traffic interruptions (e.g., accidents, pedestrian, tolling station, signal light) always occur with some probabilities resulting in complex phenomena. Perc [17] pointed out that interruptions on the lattice were due to evolutionary games. Furthermore, traffic jams become more and more serious because various traffic interruption factors often occur [18–22]. In fact, traffic interruptions always appear with some probabilities. To investigate the traffic

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http://dx.doi.org/10.1016/j.physa.2015.01.045

interruption probability explicitly on the car-following behaviors, Tang et al. [23,24] proposed a macroscopic continuum model and a car-following model with the traffic interruption probability term. Subsequently, by considering the traffic interruption probability and lane changing behaviors at the same time, Tian and Sun [25] presented a two-lane macroscopic continuum model on the basis of Tang's single lane macroscopic continuum model. Very recently, Peng et al. [26] further developed a new lattice model of traffic flow by considering traffic interruption of traffic flux. Li et al. [27] studied the effect of traffic accidents on a single-lane road with multi-slowdown sections. These results show that the traffic interruption plays important role on traffic flow. However, the traffic interruption probability of optimal current was not involved in existing traffic lattice models. In addition, Peng [28] proved that the optimal current difference plays important effect on traffic flow. Therefore, we proposed a new lattice model of traffic flow accounting for the traffic interruption probability of optimal current. The theoretic analytical and numerical simulations on the traffic stability and jamming transition will be carried out to validate our reasonable consideration.

2. The new model

The first lattice hydrodynamic model proposed by Nagatani [29,30] successfully analyzed the density wave in traffic flow based on the idea of the microscopic optimal velocity model, whose dynamic equation is shown as follows:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{1}$$

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}) \tag{2}$$

where ρ_0, ρ_j and v_j are the average density, the local density and local velocity, respectively. Subsequently, many extended lattice models [31–47] have been developed with the consideration of other factors based on the original lattice model. However, the effect of optimal current difference on traffic flow plays an important role on stability of traffic flow. Therefore, we proposed a new evolution equation as follows:

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}) + \lambda_1(\rho_0 V(\rho_{j+2}) - \rho_0 V(\rho_{j+1})) \tag{3}$$

where $\rho_0 V(\rho_{j+1})$, $(\rho_0 V(\rho_{j+2}) - \rho_0 V(\rho_{j+1}))$ and λ_1 mean the optimal current, the optimal current difference and reaction coefficient on site $j+1$ at time t, respectively. In real traffic, traffic interruption often occurs with some probabilities of some traffic interruption factors. Therefore, we develop a new lattice model by considering the interruption probability of optimal current to investigate the traffic stability and jamming transition as follows:

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}) + \lambda_1(1-p_{j+2})(\rho_0 V(\rho_{j+2}) - \rho_0 V(\rho_{j+1})) + \lambda_2 p_{j+2}(-\rho_0 V(\rho_{j+1})) \tag{4}$$

where p_{j+2} is the probability that the optimal current on site $j+2$ is interrupted; λ_2 is the reactive coefficients of the optimal current interruption. We approximately set the optimal current of the $(j+2)$th lattice as zero when it is interrupted at the probability p_{j+2}. It is to say that the optimal current of the $(j+2)$th lattice is zero with probability p_{j+2} when traffic interruption factors occur. For simplicity, the traffic interruption probability is assumed as constant, i.e., $p_{j+1} = p$. Then, the optimal current difference between the $(j+2)$th site and the $(j+1)$th site is $-\rho_0 V(\rho_{j+1})$. The optimal velocity function $V(\rho)$ is chosen as follows [29,30]:

$$V(\rho) = (v_{\max}/2)[\tanh(1/\rho - h_c) + \tanh(h_c)] \tag{5}$$

where h_c and $v_{\max}$ show the safety distance and the maximal velocity, respectively. By eliminating the speed v in Eqs. (1) and (4), we obtain the following density equation:

$$\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau\rho_0^2(1-\lambda_2 p)[V(\rho_{j+1}) - V(\rho_j)] + \lambda_1\tau\rho_0^2(1-p)[V(\rho_{j+2}) - 2V(\rho_{j+1}) + V(\rho_j)] = 0. \tag{6}$$

3. Linear stability analysis

The constant density ρ_0 and the optimal velocity $V(\rho_0)$ are assumed for the steady state of the uniform traffic flow. y_j is supposed to be a small deviation from the steady-state flow on site j.

$$\rho_j(t) = \rho_0 + y_j(t). \tag{7}$$

Substituting Eq. (7) into Eq. (6) and linearizing it, one can obtain:

$$y_j(t+2\tau) - y_j(t+\tau) + \tau(1-\lambda_2 p)\rho_o^2 V'(\rho_0)\Delta y_j(t) + \tau\lambda_1(1-p)\rho_o^2 V'(\rho_0)(\Delta y_{j+1}(t) - \Delta y_j(t)) = 0 \tag{8}$$

where $\Delta y_j = y_{j+1} - y_j$ and $V'(\rho_0) = \mathrm{d}V(\rho)/\mathrm{d}\rho|_{\rho=\rho_0}$. By expanding $y_j = A\exp(\mathrm{i}kj + zt)$, one deduces the following equation of z:

$$\mathrm{e}^{2z\tau} - \mathrm{e}^{z\tau} + (1-\lambda_2 p)\tau\rho_o^2 V'(\rho_0)(\mathrm{e}^{\mathrm{i}k} - 1) + \lambda_1 p\tau\rho_o^2 V'(\rho_0)\mathrm{e}^{\mathrm{i}k}(\mathrm{e}^{2ik} - \mathrm{e}^{\mathrm{i}k} + 1) = 0. \tag{9}$$