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A higher-order macroscopic model for bi-direction pedestrian flow

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HIGHLIGHTS

- A higher-order macroscopic model for bi-direction pedestrian flow is presented.
- The desired direction of motion for each group satisfies a reactive dynamic user equilibrium assignment pattern.
- This model can reproduce the formation, merging and disappearance of the self-organized lanes.
- A broken symmetry is also observed.

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ABSTRACT

In this paper, a higher-order macroscopic model for bi-direction pedestrian flow is presented to investigate macroscopic crowd patterns of bi-direction pedestrian movement. For each group, the macroscopic model consists of two-dimensional Euler equations with relaxation and an Eikonal-type equation. The magnitude of the desired velocity is described by an empirical relation between density and speed, while its direction is chosen to minimize the total instantaneous walking cost which satisfies a reactive user equilibrium assignment pattern. The dynamic model is numerically solved by a splitting technique and a cell-centered finite volume method on an orthogonal grid. Typical numerical examples of bi-direction pedestrian flow walking in a channel are designed to validate the rationality of the dynamic model and the effectiveness of the numerical algorithm.

1. Introduction

Research on pedestrian crowd characteristics and behaviors has recently become popular and attracted considerable attention. The behavioral characteristics of pedestrian movement involve flow rate, velocity, density and the fundamental relationships between them. Understanding pedestrian movement plays a significant role in the design, management and safety of transportation facilities, pedestrian walkways and public transport intersections.

Crowd dynamics is very complex as problems typically arise due to counter flows, bottlenecks, or intersecting flows in pedestrian traffic [1,2]. Up to now, many observations on pedestrian flow problems, including various self-organization phenomena, have been made by experimental studies and model simulations. For instance, experimental studies of pedestrian movement at typical elements of buildings and pedestrian facilities have displayed many quantitative features of the

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interaction laws among individuals [2–7], e.g. the walking speed and fundamental diagram. Some features have been successfully cast into mathematical equations to describe pedestrian dynamics, e.g. fluid dynamic models [8–10]. However, most real-life experiments for the investigation of crowd motion are extremely difficult or even impossible. Therefore, a number of simulation models for pedestrian flows, microscopic [11–18] vs. macroscopic [19–24], have been presented to understand the underlying dynamics of crowd behaviors. Diverse bi-direction pedestrian flows (or two streams of pedestrians moving in intersecting directions) as typical pedestrian flow phenomena have been studied extensively. Based on some microscopic models, such as social force models [12,13] and cellular automata models [16,17], phase transitions (e.g. the jamming and freezing transitions when pedestrian density increases) and self-organization phenomena (e.g. lane formation, oscillations at bottlenecks and dynamics at intersections) are investigated. The self-organized collective behaviors occurring in bi-direction pedestrian flows may lead to unexpected obstructions due to mutual disturbances of pedestrian flows. However, they can be utilized for an optimization of pedestrian flows by reducing frictional effects, local accelerations, energy consumptions and walking delays [1,25].

In this paper, a higher-order macroscopic model for bi-direction pedestrian flow, which is extended from a single pedestrian type higher-order model [23], is presented. Pedestrians walking rightward and leftward in a two-dimensional (2D) continuous walking facility are classified into two groups. For each group of pedestrian flow, the macroscopic model consists of 2D Euler equations with relaxation and an Eikonal-type equation. The Eikonal-type equation is introduced to describe the desired direction of motion that minimizes the total instantaneous walking cost from origin to destination, which describes a reactive user equilibrium (RUE) assignment pattern. The magnitude of the desired velocity is described by an empirical relation between density and speed. The algorithm for the dynamic model is composed of a splitting technique and a cell-centered finite volume method (FVM) on an orthogonal grid. Numerical results show that this higher-order model has the ability of capturing fundamental properties of bi-direction pedestrian movement. Based on this model, the formation, merging and dissipation of self-organized lanes and a broken symmetry on large scales are also observed.

The outline of this paper is as follows. In the next section, the mathematical model for bi-direction pedestrian flow is described in detail. Section 3 gives a numerical algorithm for the model. The numerical results are presented in Section 4. The last section includes some concluding remarks.

2. Problem formulation

We assume that heterogeneous pedestrians in a 2D continuous walking facility Ω (in m^2) are classified into two groups by their different purposes of travel. Group $l(r)$ express pedestrians marching towards the right (left) exit of the facility. Γ (in m) is the boundary of Ω, Γ_o^k (in m) is the original segment through which Group k enter the facility, Γ_d^k (in m) is the destination segment through which Group k leave the facility, and Γ_h^k (in m) is the wall segment through which nobody in Group k is allowed to enter or leave the facility. T_0 (in s) is the time horizon. Here, $\Gamma = \Gamma_o^k \bigcup \Gamma_d^k \bigcup \Gamma_h^k$, $k \in \{l, r\}$, $(x, y) \in \Omega$ and $t \in T_0$. The density of Group k at location (x, y) and time t is denoted as $\rho^k(x, y, t)$ (in ped/m^2) and the average velocity of Group k is represented as $\mathbf{v}^k(x, y, t) = (u^k(x, y, t), v^k(x, y, t))$ (in m/s), where $u^k(x, y, t)$, $v^k(x, y, t)$ are speeds in the x- and y-directions, respectively. Let $\vec{\nu}^k = (\nu_x^k, \nu_y^k)$ be a unit vector which represents the desired motion direction of Group k and $\Psi(x, y, t)$ be the intersecting angle between Group l and Group r at location (x, y) and time t.

2.1. The dynamic model for bi-direction pedestrian flow

A crowd of heterogeneous pedestrians in a 2D continuous domain can be considered and analyzed as a homogeneous fluid [19] and a higher-order macroscopic model for unidirectional pedestrian flow is formulated as follows [23].

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\ \dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + c^2(\rho)\dfrac{\nabla \rho}{\rho} = \dfrac{U_e(\rho)\vec{\nu} - \mathbf{v}}{\tau}, \end{cases} \tag{1}$$

where $\rho(x, y, t)$ denotes the density of the unidirectional pedestrian flow; $u(x, y, t)$, $v(x, y, t)$ are the average speeds of pedestrian motion in the x- and y-direction, respectively; $U_e(\rho)$ is the speed–density relationship; $c(\rho)$ (in m/s) is the equivalent traffic sonic speed describing the propagating speed of small perturbation in pedestrian flow; $\vec{\nu}$ represents a desired direction of motion; and τ (in s) is a characteristic relaxation time. Compared to the first-order macroscopic pedestrian flow model [22], this higher-order model is able to reproduce complex crowd dynamics such as stop-and-go waves and clogging at bottlenecks that are observed in the experiments [24].

We consider heterogeneous pedestrians in Ω are characterized by their different travel destinations. The model for unidirectional pedestrian flow (1) is extended to describe macroscopic features and path choice behaviors of bi-direction pedestrian flow. The higher-order dynamic model for non-equilibrium bi-direction pedestrian flow is written as

$$\begin{cases} \dfrac{\partial \rho^k}{\partial t} + \nabla \cdot (\rho^k \mathbf{v}^k) = 0, \\ \dfrac{\partial \mathbf{v}^k}{\partial t} + (\mathbf{v}^k \cdot \nabla)\mathbf{v}^k + c^2(\rho)\dfrac{\nabla \rho}{\rho^k} = \dfrac{U_e^k\vec{\nu}^k - \mathbf{v}^k}{\tau}, \end{cases} \tag{2}$$