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Effects of time-delay in stationary properties of a logistic growth model with correlated noises☆

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ABSTRACT

A time-delayed tumor cell growth model with correlated noises is investigated. In the condition of small delay time, the stationary probability distribution is derived and the stationary mean value ($\langle x\rangle_{st}$) *and normalized variance* λ_2 of the tumor cell population and state transition rate (κ) between two steady states are numerically calculated. The results indicate that: (i) The delay time (τ) enhances the coherence resonance in $\langle x\rangle_{st}$ as a function of the multiplicative noise intensity (D) and increases $\langle x\rangle_{st}$ as a function of the additive noise intensity (α), i.e., τ enhances fluctuation of the system, however, the strength (λ) of correlations between multiplicative and additive noise plays a contrary role to τ on these; (ii) τ enhances the coherence resonance in κ as a function of D and increases κ as a function of α, i.e., τ speeds up the rate of state transition, however, λ also plays a contrary role to τ on these.

1. Introduction

Logistic growth is one of the most popular equations not only in mathematical ecology but also in other applications. First introduced by Verhulst for saturated proliferation of a single-species [1], it has been extended to include spatial dynamics by Fisher [2] and by Kolmogoroff et al. [3]. It is now one of the classical examples of self-organization in many natural and artificial systems [4]. In this paper we focus on time delay effects on the basis of the logistic growth equation. In particular, this equation was proposed to describe the growth of the Ehrlich ascities tumor (EAT) in a mouse [5]. It appears that even such a simple ordinary differential equation can be used to model such a complicated process.

The equation of the logistic growth model reads

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax - bx^2. \tag{1}$$

Here x is the tumor mass (or denotes tumor cell population), so is confined to positive real numbers, a is the cell growth rate and b is the cell decay rate. Previously, much attention has been paid to the statistical properties of the tumor cell growth model with correlated noises [6–11]. It was found that the correlations between noises essentially affect the stationary and transient properties of the model. However, in previous works, the members of the population were assumed to react instantaneously to any change in the environment; a time delay was not included in the logistic growth model. For population dynamics, there should be a reaction time of the population to environmental constraints in the process of the population evolution [12,13]. Time delay can play a crucial role in the modeling of biological processes [14], especially on the cellular level, which is extremely important in tumor growth modelling [15,16]. Thus, the logistic growth model with a time delay driven by cross-correlation noises should be investigated.

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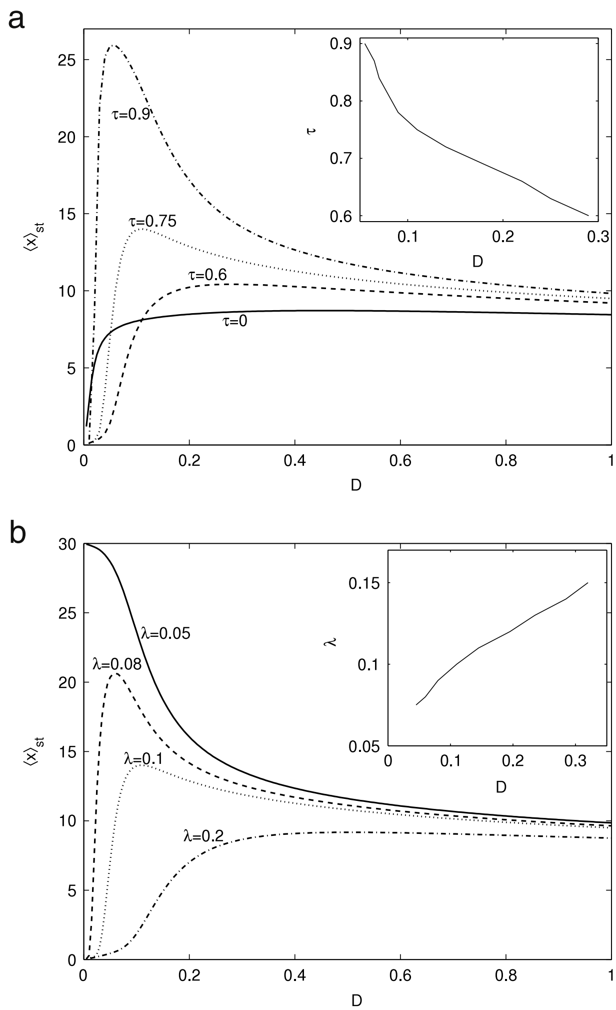

Fig. 1. The stationary mean value of the tumor cell population $\langle x\rangle_{st}$ as a function D calculated from Eq. (14) with $a = 1, b = 0.1, \alpha = 3, \tau' = 0.2$. (a) $\lambda = 0.1$. (b) $\tau = 0.75$. The insets show the D value for maximum $\langle x\rangle_{st}$ vs. τ and λ respectively.

In this paper, we investigate the effects of time delay in a logistic growth model with the combination of correlated noises with nonzero correlation time and time delay. In Section 2, we derive the approximate Fokker–Planck equation and the stationary probability distribution (SPD) of the model, and calculate the mean ($\langle x\rangle_{st}$) of the population and the state transition rate (κ) between two stable states of the system. Section 3 consists of a discussion and the conclusion to the paper.

2. The statistical properties of the system with delay

The Langevin equation of the logistic growth model subjected to correlated noises reads [6,7]

$$\frac{dx}{dt} = ax - bx^2 + x\xi(t) - \eta(t), \tag{2}$$

where $\xi(t)$ and $\eta(t)$ are the cross-correlated Gaussian white noises, originating from the fluctuations of some external factors, such as temperature, drugs, radiotherapy and so on, and have the following statistical properties: [7]

$$\langle \xi(t)\rangle = \langle \eta(t)\rangle = 0, \tag{3}$$

$$\langle \xi(t)\xi(t')\rangle = 2D\delta(t - t'), \tag{4}$$

$$\langle \eta(t)\eta(t')\rangle = 2\alpha\delta(t - t'), \tag{5}$$

and

$$\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{D\alpha}}{\tau'}\exp\left[\frac{-|t - t'|}{\tau'}\right] \rightarrow 2\lambda\sqrt{D\alpha}\delta(t - t') \quad \text{as } \tau' \rightarrow 0, \tag{6}$$