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Constrained information minority game: How was the night at El Farol?

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ABSTRACT

We introduce a variation of the El Farol Game in which the only players who surely know the outcome of the last turn of the game are those who actually attended the bar. Other players may receive this information with reduced probability. This information can be transmitted by another player who actually attended the bar in the last turn of the game or from the media. We show that since this game is not organized around the socially optimal point, arbitrage opportunities may arise. Therefore, we study how these opportunities can be exploited by an agent. An interesting application of this model is the market of goods being auctioned, such as cars being repossessed. The results obtained here seem to closely reflect the dynamics of this market in Brazil.

1. Introduction

In recent years, a promising area of research has been the dynamics of populations of agents and their collective behavior when competing for limited resources [1,2]. One specific example of this class of model is the so-called Minority Game (MG) introduced in Ref. [3] as a simplification of Arthur's El Farol Bar [4] attendance problem, which is one of the simplest complex systems that belong to this class. The Standard Minority Game (SMG) has been very well-studied—a review of these studies can be found in Refs. [2,5,6].

Since in real life agents are usually connected in social networks [7,8], several works [9–20] have considered a variation of the SMG where the networked agents who play the game access different bits of information from their neighbors. These attempts have successfully modeled the influence of local information on the dynamics of the minority game.

Another variation of the SMG has considered that the histories of the individual agents who play the game are different [21–23]. This approach has been used to analyze the effect of the amount of information available to agents on their performance.

In this paper we consider the scenario where the only players who surely know the outcome of the last round of the El Farol Game [4] are those who actually attended the bar. Other players may receive this information with reduced probability. In fact, they only access this information directly from players who actually attended the bar in the last turn of the game or from the media. As observed in real life, especially in the financial markets, outcomes of previous rounds of a game are likely disseminated after extremes, when people either celebrate good deals or complain about bad results. The media also

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emphasize these extreme good or bad performances, which increases the odds of people having access to the results. Since information about the outcome of the game is constrained to a part of the agents, we call this game "Constrained Information Minority Game" (CIMG).

In the CIMG, one can observe the two elements presented above, namely agents having access to different bits of information [9–20] and agents that make decisions based on different histories [21–23].

One interesting application of this game is as a model for the auctioning of goods, such as cars, which have been repossessed. According to the Brazilian law, personal belongings acquired with bank financing must be auctioned when people do not pay their loans.

There are some interesting characteristics of this market:

(1) Cars can be bought at auctions, but they can also be purchased at other markets.
(2) Cars auctions are conducted quite frequently.
(3) The results of a given auction are only precariously announced.
(4) The interested population only gets information about the outcome (price and quality of the objects to be auctioned) of previous auctions by means of reports from those who actually went to the auction. This information is usually shared only when people are complaining from the bad deals obtained at a previous auction or reporting good ones. The strength of this information is usually correlated to deviations in the outcome of that auction from the outcome of a typical auction.
(5) Auctions of repossessed cars closely resemble the minority game, specially to the El Farol attendance problem. If fewer people attend an auction, people who attend are likely to get better deals. On the other hand, if too many people attend the auction, the prices of the goods auctioned are high. Therefore, it is better to stay at home.

In order to consider the scenario of the auction of repossessed cars, we have modified the standard minority game to take the characteristics of this market into consideration.

This paper proceeds as follows. In Section 2 we precisely define the setup of the CIMG. In Section 3 we show the results. Finally, in Section 4 we present some final remarks.

2. Setup of the CIMG

The framework for this kind of game is quite similar to the SMG [3]. Agents compete with each other in an attempt to be on the side of the minority. In each turn t, each agent i based on his highest scored strategy chooses between two opposite actions $a = \pm 1$. A strategy in a given time is considered successful if it correctly predicts the minority side. The strategies with highest scores are those which were the most successful in the previous turns of the game. In this paper, $a = 1$ can be interpreted as the decision to go to the bar or to participate in the car auction and $a = -1$ can be interpreted as the opposite. Each specific strategy s_i defines a specific action $a^{\mu}_{s_i,i}$ for each state μ observed by the agent. Since M represents the history of the game, it is clear that the number of possible states is $P = 2^M$. Once all agents have defined their actions for round t, the sum of these actions defines the outcome of that round $A(t) = \sum_i a^{\mu(t)}_{s_{i(t)},i}$.

The rule used to update the scores of the strategies is presented below. The score $U_{s,t}$ for each strategy is initiated with 0. When there is a tie among possible strategies, the agent chooses randomly between them, using the same probability for each strategy. As usual in the SMG, strategies are updated based on the following function

$$U_{s,t}(t+1) = U_{s,t}(t) - a^{\mu(t)}_{s,i} A(t). \quad (1)$$

The action $a^{\mu}_{s_i,i}$ of each player i, linked to each strategy s_i for each state μ is initially chosen from the possible values of -1 and $+1$ with equal probability.

The difference between the CIMG and the SMG is that in the former, for each turn of the game there are two types of agents: Type 1 agents who attended the bar in the previous turn of the game and have access to the outcome of the game. Type 2 agents who did not attend the bar in the previous round of the game and access the outcome of the game with probability p, endogenously defined as a function of the attendance of the auction in the previous turn of the game.

While type 1 agents update their strategies as agents in the SMG, type 2 agents who did not receive information about the outcome of the game are not able to update their strategies nor to update their own history. Therefore, they have a different view from that of the ordinary global history.

2.1. The dynamics of the probability p

As mentioned above, some of the N agents do not know the outcome of the last turn of the game. If an agent plays $a(t) = 1$ in the last round of the game, he knows the outcome of the game with $p = 1$. Otherwise, he knows the outcome of the game with a smaller probability $p = \max\left\{\frac{|A(t)|}{N}, p_{\min}\right\}$. $p_{\min}$ refers to the minimal probability of a type 2 agent accessing the outcomes of the game. Fig. 1 presents p as a function of $A(t)$.

It is easy to understand this idea if one considers the problem of the El Farol Bar. Consider an agent who went to the bar in the last turn of the game. This agent knows with probability 1 how good the bar was that night. Those agents who did