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Physica A 376 (2007) 565–572
PHYSICA A
www.elsevier.com/locate/physa

The stability of electricity prices: Estimation and inference of the Lyapunov exponents ☆

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Received 10 July 2006; received in revised form 14 September 2006
Available online 27 October 2006

Abstract

The aim of this paper is to illustrate how the stability of a stochastic dynamic system is measured using the Lyapunov exponents. Specifically, we use a feedforward neural network to estimate these exponents as well as asymptotic results for this estimator to test for unstable (chaotic) dynamics. The data set used is spot electricity prices from the Nordic power exchange market, Nord Pool, and the dynamic system that generates these prices appears to be chaotic in one case since the null hypothesis of a non-positive largest Lyapunov exponent is rejected at the 1 per cent level.

Keywords: Feedforward neural network; Lyapunov exponents; Nord pool; Spot electricity prices; Stochastic dynamic system

1. Introduction

The aim of this paper is to illustrate how the stability of a stochastic dynamic system is measured using the Lyapunov exponents. Specifically, we use a feedforward neural network to estimate these exponents as well as asymptotic results for this estimator to test for unstable (chaotic) dynamics, where a positive exponent is an operational definition of chaos. The data set used is spot electricity prices from the Nordic power exchange market, Nord Pool.

The estimation of the Lyapunov exponents using a feedforward neural network can be found in earlier studies such as Dechert and Gencay [1], Gencay and Dechert [2], McCaffrey et al. [3] and Nychka et al. [4]. The estimation of these exponents has been proved to be quite accurate when applying chaotic series with additive noise in simulations. However, the statistical properties of the Lyapunov exponent estimator were

☆ Earlier drafts of this paper have benefited from presentations at the "14th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics" in St. Louis, USA, March 24–25, 2006, and at "Nonlinear Dynamical Methods and Time Series Analysis" in Udine, Italy, August 30–September 1, 2006. We are also grateful to an anonymous referee for comments that improved the paper. Of course, the usual disclaimer applies.
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0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.10.016

unknown until Shintani and Linton's 2004 paper (see Ref. [5]), and without the statistical distribution for this estimator, no statistical conclusion can be drawn on the dynamic structure of the empirical data.

This paper applies the statistical distribution derived in Shintani and Linton [5] to test the stability of spot electricity prices from Nord Pool, and the stochastic dynamic system that generates these prices appears to be chaotic in one case since the null hypothesis of a non-positive largest Lyapunov exponent is rejected at the 1% level.

The rest of this short paper is organized as follows: the Lyapunov exponents are in focus in Section 2, the empirical illustration is carried out in Section 3, and Section 4 concludes the paper with a remark.

2. The Lyapunov exponents

The aim of this section is fourfold: (i) to define the Lyapunov exponents of a stochastic dynamic system; (ii) to motivate why these exponents provide a measure of the stability of a stochastic dynamic system; (iii) to demonstrate how the Lyapunov exponents can be estimated from time series data; and (iv) to demonstrate how hypothesis tests of these exponents can be constructed.

2.1. Definition of the Lyapunov exponents

As argued in Bask and de Luna [6,7], and to be further explained in Section 2.2, the Lyapunov exponents can be used in the determination of the stability of a stochastic dynamic system. Specifically, assume that the stochastic dynamic system, $f : \mathbb{R}^n \to \mathbb{R}^n$, generating, for example, asset returns is

$$S_{t+1} = f(S_t) + \varepsilon_{t+1}^s, \tag{1}$$

where S_t and ε_t^s are the state of the system and a shock to the system, respectively, both at time $t \in [1, 2, \ldots, \infty]$. For an n-dimensional system as in (1), there are n Lyapunov exponents that are ranked from the largest to the smallest exponent:

$$\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n, \tag{2}$$

and it is these exponents that provide information on the stability properties of the dynamic system f in (1).

Now, how are the Lyapunov exponents in (2) defined? Temporarily, assume that there are no shocks to the dynamic system f in (1), and consider how the system amplifies a small difference between the initial states S_0 and S_0':

$$S_j - S_j' = f^j(S_0) - f^j(S_0') \simeq Df^j(S_0)(S_0 - S_0'), \tag{3}$$

where $f^j(S_0) = f(\cdots f(f(S_0))\cdots)$ denotes j successive iterations of the dynamic system starting at state S_0, and where Df is the Jacobian of the system:

$$Df^j(S_0) = Df(S_{j-1})Df(S_{j-2})\cdots Df(S_0). \tag{4}$$

Then, associated with each Lyapunov exponent, λ_i, $i \in [1, 2, \ldots, n]$, there are nested subspaces $U^i \subset \mathbb{R}^n$ of dimension $n + 1 - i$ with the property that

$$\lambda_i \equiv \lim_{j\to\infty} \frac{\log_e \| Df^j(S_0)\|}{j} = \lim_{j\to\infty} \frac{1}{j}\sum_{k=0}^{j-1} \log_e \|Df(S_k)\|, \tag{5}$$

for all $S_0 \in U^i - U^{i+1}$. Due to Oseledec's multiplicative ergodic theorem, the limits in (5) exist and are independent of S_0 almost surely with respect to the measure induced by the process $\{S_t\}_{t=1}^{\infty}$.[1] Then, allow for shocks to the dynamic system f in (1), meaning that the aforementioned measure is induced by a stochastic process.

[1] See Guckenheimer and Holmes [8] for a careful definition of the Lyapunov exponents and their properties.