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Fluctuations in driven systems

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HIGHLIGHTS

- A formalism for describing systems in local thermal equilibrium is described.
- Two building blocks, describing a drive and dissipation, are introduced and studied.
- Their time evolution and the steady-state properties are described.
- More complicated setups can be constructed from the building blocks.
- The formalism is used to understand correlations in conserving transport channels.

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ABSTRACT

The lecture notes give an introduction to some aspects of fluctuations in driven systems. First, a particularly straightforward derivation of fluctuation theorems and their relation to linear response will be given. Being particularly simple the derivation gives clear limitations on the regime of validity of the theorems. Second, an introduction to systems in local thermal equilibrium, in time and space, will be given. To this end, two building blocks will be studied. One a system which is driven by some cyclically changing external potential. The other consisting of two systems, at different temperatures, which are equilibrating. Both building blocks display an interesting time evolution. For example, the statistical properties of fluctuations of the energy in driven isolated systems generally fall into two types of distinct behaviors. Following this a systematic approach for describing more complicated systems will be outlined by studying other setups. Specifically, driven-dissipative systems and systems driven by two external baths at different temperatures. The formalism will be used to elucidate, in a minimal model, the origin and sign of correlation in a boundary driven system.

1. Introduction

Statistical mechanics provides a powerful tool for calculating both fluctuations and typical behaviors (usually encoded in average quantities) for systems in thermal equilibrium [1]. For example, it is straightforward to calculate the mean energy and the energy fluctuations of, say, an ideal gas connected to a heat bath. The mean energy, in three-dimensions, is given by $3NT/2$ and the variance of the energy fluctuations is given by T^2C. Here N is the number of particles in the gas, T is the temperature of the system and the bath, $C = 3N/2$ is the specific heat and the Boltzmann constant has been set to one.

A much broader question concerns systems away from thermal equilibrium. Loosely speaking these can be classified into two broad classes: (a) systems which are evolving towards a steady-state (which may or may not be a thermal equilibrium state), and (b) systems which are in a non-equilibrium steady-state so that their statistical properties do not change in time and are in general not described by a Gibbs distribution.

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As an example of a class (a) system consider, again, an ideal gas which is thermally isolated and connected to a piston. Given that the gas starts at an initial energy E, it is natural to ask how its energy evolves as the piston is displaced. Equilibrium statistical physics provides an answer in the reversible limit [1]. In that limit the piston moves so slowly that the gas is able, loosely speaking, to sample all of phase space at each piston position and only thermodynamic averages govern the energy changes. Therefore, the energy changes are deterministic. In particular, one can trace back the evolution of the system and returns it to its exact macroscopic initial condition. However, when the process is not reversible the outcome of different experiments, even if the protocol which pulls the piston is identical, is random. The system is not guaranteed to reach the same macroscopic final configuration. In general very little is known about the probability distribution which characterizes such processes. The answer seems hard as it depends on the full dynamical properties of the system. In spite of this, recently a series of sum-rules and symmetries, which the probability distributions must satisfy, have been uncovered [2–13]. They go under the general name of fluctuation-theorems. Unfortunately, outside the usual linear response regime these contain very little information about the actual structure of the probability distribution. This discussion follows similarly for other class (a) systems. For example, instead of driving the gas at an initial temperature T_a with a piston one could couple it to another system at temperature T_b and ask how the energy of the gas evolves in time towards the equilibrium state at which the two temperatures equilibrate. Again, besides the statement that fluctuation theorems hold very little is known about actual probability distributions which characterize the process.

As an example of class (b) systems, consider again a gas which is driven, say by a piston at one end, and connected to a bath at temperature T_b at the other end. If the piston is moved in some cyclic manner, and we observe the system on time scales much longer than those of a cycle, the system will eventually reach a steady-state where the average amount of energy injected by the piston is equal to the average amount of energy dissipated into the bath. This non-equilibrium steady-state in general will not be characterized by a Gibbs distribution and one could ask about its statistical properties. Similarly, instead of connecting the system to a drive at one end and a bath at the other, one could connect the system to two baths at different temperatures. Again the resulting steady-state will in general not be a Gibbs distribution. Our understanding of the steady-state of non-equilibrium systems is much poorer than the equilibrium counterpart. This can again be traced back to the dependence of the probability distribution on the details of the dynamics. There are, however, classes of non-equilibrium systems at steady-state for which much can be said. For example, there has been much understanding of fluctuations in non-equilibrium diffusive systems [14] (and the very closely related problem of hydrodynamic fluctuations [15,16]). It is now know that out of equilibrium, when a system with conserving diffusive dynamics is connected to external reservoirs that drive it out of equilibrium generic long-ranged correlations emerge outside the linear response regime [17–21]. There are other examples of non-equilibrium systems in a steady-state where much progress has been made, for example, coarsening systems [22], driven-diffusive system in the strong bias regime [23] or systems which undergo a transition into an absorbing state [24]. These and others fall outside the scope of these lecture notes and will not be discussed.

The purpose of these lecture notes is to consider some aspects of the statistical properties of fluctuations in class (a) and class (b) systems. First, an overview of fluctuation theorems will be given. As stated above, these contain little information on actual probability distribution unless changes are within (the standard) linear-response regime. This statement along with the formal derivation of linear-response relations will be made explicit. The main part of the lecture notes will be devoted to describing both class (a) and (b) systems where a systematic approach for calculating statistical properties exists. The main assumption used is that of local equilibrium, in time and space, and the focus will be on studying energy fluctuations in such systems which are not captured by equilibrium reversible dynamics. In developing the framework for describing systems in local thermal equilibrium the fluctuation relations, discussed initially, will be handy — in particular the linear response relations which are derived from them. Note that the discussion will mostly focus on energy probability distributions but the results can easily cast in terms of other quantities (for example, particle number fluctuations).

The assumption of local equilibrium is, of course, by no means new or novel and has been employed for many years to describe a host of systems. For example, most of the work on continuum diffusive systems or hydrodynamic fluctuations uses the same approach [15,16]. The presentation used here is, however, distinct from previous ones and will include some more recent results [25–27]. First, two building blocks will be described. The first is a system which is driven by some cyclically changing external potential. The second are two systems, at different temperatures, which are equilibrating. It will be shown that even these simple building blocks display interesting behavior. In particular, the time evolution of fluctuations will be shown to be rather rich. For example, it will be shown that the statistical properties of driven isolated systems generally fall into two types of distinct behaviors. Following this a systematic approach for describing more complicated systems will be outlined by studying two simple general setups: driven-dissipative systems and systems driven by two external baths at different temperatures. As will become evident the formalism can be easily extended to other much more complicated scenarios. As an illustration it will be used to shed light on the origin and different signs of correlation functions in continuum models of boundary driven systems.

2. Fluctuation theorems and linear response

Fluctuation theorems are sum rules and symmetries which apply to driven systems. By now there are many different fluctuation theorems that have been derived and undoubtedly many more can be found. Typically fluctuation theorems are derived for a system coupled to a infinite bath. This also implies that any work done on the system is negligible compared to the energy of the bath.