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# Exact and numerical solitons with compact support for nonlinear dispersive $K(m,p)$ equations by the variational iteration method

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**Abstract**

We implemented a variational method for approximating the solution of the nonlinear dispersive $K(m,p)$ type equations. By using this scheme, the explicit exact solution is calculated in the form of a quickly convergent series with easily computable components. To illustrate the application of this method, numerical results are derived by using the calculated components of the variational series. The obtained results are found to be in good agreement with the exact solution.


*Keywords:* Nonlinear dispersive $K(m,p)$ equations; Variational iteration method; Lagrange multiplier

## 1. Introduction

The variational iteration method [1–7] is a powerful method to investigate approximate solutions or even closed form analytical solutions of nonlinear evolution equations. In addition, no linearization or perturbation is required by the method. Recently, the method has been applied to investigate many nonlinear partial differential equations and autonomous and singular ordinary differential equations such that solitary wave solutions, rational solutions, compacton solutions and other types of solution were found [1–19]. The variational iteration method has many merits and has much advantages over the Adomian decomposition method [13–17].

Drăgănescu and Căpălnăsan [10] applied the variational iteration method to a nonlinear elastic model describing the acceleration of the relaxation process in the presence of the vibrations. The combination of a perturbation method, variational iteration method, method of variation of constants and averaging method to establish an approximate solution of one degree of freedom on weakly nonlinear systems was proposed in Ref. [11]. The application of the variational iteration method to nonlinear fractional differential equations and Helmholtz equation can be found in Refs. [7,12,13]. This method is used for solving nonlinear dispersive equations in Ref. [14]. In Ref. [15] the applications of the present methods to some nonlinear partial

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differential equations are provided. Moreover, the method was successfully applied to delay differential equations in Ref. [2], to autonomous ordinary differential systems [4], and other fields [16,17].

In the past decades, directly seeking for exact solutions of nonlinear partial differential equations has become one of the central themes of perpetual interest in Mathematical Physics. Nonlinear wave phenomena appear in many fields, such as fluid mechanics, plasma physics, biology, hydrodynamics, solid state physics, and optical fibers. These nonlinear phenomena are often related to nonlinear wave equations. In order to understand better these phenomena as well as further apply them in the practical life, it is important to seek their exact solutions. Many powerful methods had been developed such as Backlund transformation [18,19], Darboux transformation [20], the inverse scattering transformation [21], the bilinear method [22], the tanh method [23,24], the sine–cosine method [25,26], the homogeneous balance method [27], the Riccati method [28], the Jacobi elliptic function method [29], the extended Jacobi elliptic function method [30], etc.

In the well-known Korteweg–de Vries (KdV) equation

$$u_t - auu_x + u_{xxx} = 0, \tag{1}$$

the nonlinear term $uu_x$ causes the steepening of the wave form. On the other hand, the dispersion term $u_{xxx}$ in this equation makes the wave form spread. Due to the balance between this weak nonlinearty and dispersion, solitons exist [31].

The genuinely nonlinear dispersive equation $K(m,p)$, which generalizes the KdV equation (1), is given by

$$u_t + a(u^m)_x + (u^p)_{xxx} = 0, \quad m,p \geqslant 1, \tag{2}$$

see Rosenau and Hyman [32]. The convection term of $K(m,p)$ is obviously the nonlinear part of it. Additionally, by this reason the scattering term in Eq. (2) is genuinely nonlinear. In Refs. [30,32], the authors mentioned that the important interaction in between nonlinear convection with real nonlinear scattering generates solitary waves with exact compact support which are called compactons. Compacton is a soliton solution which have finite wave length or free of exponential wings. Unlike soliton that narrows as the amplitude increases, the compactons width is independent of its amplitude. The solution of Eq. (2) is also very interesting because of the local nature by which it can serve as the reflection of a wide range of extraordinary nature [33].

The aim of this paper is to extend the variational iteration method of He [1–7] to derive the numerical and exact compacton solutions of the nonlinear dispersive $K(m,p)$ equation (2) subject to the initial condition:

$$u_t + (u^m)_x + (u^p)_{xxx} = 0, \quad m > 1, \ \ 1 \leqslant p \leqslant 3, \tag{3a}$$

$$u(x,0) = f(x). \tag{3b}$$

## 2. Variational iteration method

To illustrate the basic concepts of the variational iteration method, we consider the following differential equation:

$$Lu + Nu = g(x,t), \tag{4}$$

where $L$ is a linear differential operator, $N$ a nonlinear operator and $g(x,t)$ an inhomogeneous term.

According to the variational iteration method, we can construct a correct functional as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^x \lambda\{Lu_n(x,\tau) + \tilde{N}u_n(x,\tau) - g(x,\tau)\}\,\mathrm{d}\tau, \quad n \geqslant 0, \tag{5}$$

where $\lambda$ is a general Lagrangian multiplier [34] which can be identified optimally by the variational theory [36–38], the subscript $n$ denotes the $n$th-order approximation, and $\tilde{u}_n$ is considered as a restricted variation [34,35], i.e., $\delta\tilde{u}_n = 0$.

To illustrate the above theory, we implement the variational iteration method for finding the exact solution of the nonlinear dispersive $K(m,p)$ equation (2) subject to (3). This problem will be handled more easily,