Quantitative agent-based firm dynamics simulation with parameters estimated by financial and transaction data analysis

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Abstract

Firm dynamics on a transaction network is considered from the standpoint of econophysics, agent-based simulations, and game theory. In this model, interacting firms rationally invest in a production facility to maximize net present value. We estimate parameters used in the model through empirical analysis of financial and transaction data. We propose two different methods (*analytical method* and *regression method*) to obtain an interaction matrix of firms. On a subset of a real transaction network, we simulate firm's revenue, cost, and fixed asset, which is the accumulated investment for the production facility. The simulation reproduces the quantitative behavior of past revenues and costs within a standard error when we use the interaction matrix estimated by the regression method, in which only transaction pairs are taken into account. Furthermore, the simulation qualitatively reproduces past data of fixed assets.
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1. Introduction

The study of firm dynamics and industry structure has a long history (for reviews, see Ref. [1]). Industrial organization literature, in particular, has long been focused on two problems: first, the distribution shape of firm size and second, the distribution shape of growth rate and relation of growth rate and firm size. Regarding these problems, recent studies of econophysics clarified that previously obtained controversial results originated in the analysis of imperfect data. Three fundamental facts were obtained through analyses of exhaustive data from European and Japanese firms (Refs. [2–5] and references in them). These fundamental facts are: (1) the upper tail of the distribution of firm size can be fitted with a power law (Pareto–Zipf Law),

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(2) in the power-law region the growth rate of each firm is independent of the firm size (Gibrat's Law), and (3) outside the power-law region the Gibrat's Law dose not hold.

Before those fundamental facts were obtained, many models were proposed to explain the power-law distribution of wealth and income, e.g. stochastic multiplicative processes without interactions (for example, Refs. [6–8]), and those with interactions (for example, Refs. [9–16]). Though these models did succeed in reproducing the power-law distribution, they are far from a realistic model of firms.

There are three reasons why previous models are not applicable to the real business world. The first reason is concerned with parameters controllable by managers. The distribution of random variables used in these models i.e., mean value and variance, can be controlled to obtain the desired power-law distributions of wealth and income. However, in real business, managers of firms cannot directly control wealth and income. The controllable parameter for managers is investment to the production facility. The second reason concerns with business-to-business transactions. In real business, managers of the firm determine the investment to their production facilities based on forecasted business-to-business transactions. Thus, it is important to incorporate a realistic network of the business-to-business transactions into the model. However, previous models have not seriously considered the transaction network. The third reason has to do with a quantitative description of firm activity. Previous studies have not considered a quantitative description of firm activity, but have concentrated only on the qualitative aspect of distribution shape. A quantitative description is needed in order to apply a model to a real business. Thus, model parameters have to be estimated through empirical data analysis.

Hence, it is essential to develop a model that can capture various changes in the business environment by considering the activities of interacting firms, in order to obtain a model applicable to real business. The purpose of this article is to propose such a model. The important features of our model can be briefly summarized as follows: (1) managers of firms can control decisions for investment in the production facility, (2) a transaction network extracted through analysis of transaction data is considered, and (3) model parameters, including interaction parameters, are estimated through analysis of financial and transaction data.

This article is organized as follows. In Section 2, an agent-based model of interacting firms is proposed, considering both of production and transaction. In Section 3, we take a small subset of a real tran saction network through empirical analysis of the real data. For the transaction network, we estimate model parameters. In Section 4, a numerical simulation is made for the transaction network and the obtained revenues, costs, and fixed assets of firms are compared with past data. Section 5 is devoted to a summary.

2. Model of interacting firms

In this section, we explain the model of interacting firms. In Section 2.1, we assume Gibrat's Law, and propose a one-body equation for the growth rate of revenue. In Section 2.2, we extend this equation to an N-body system, which contains mean value and standard deviation of stochastic variables. In Section 2.3, we replace the mean value of stochastic variables to the realized solution at the Nash equilibrium in game theory. From these processes, we obtain the N-body equation for revenue in Section 2.4. In a subsection, using the production function, we describe revenue from fixed assets, which is a variable controllable by managers and represents accumulated investment for the production facility. In Section 2.5, we consider this upper limit of revenues from the law of demand and supply on the transaction network. The solution at the Nash equilibrium is obtained by the backward induction, as explained in Section 2.6.

2.1. Definition of the one-body equation

We denote a revenue of ith firm at time t as $R_i(t)$, where i runs from 1 to N. Here, N is the number of firms which construct a transaction network. If we assume Gibrat's Law for $R_i(t)$, then the growth rate of revenue $X_i(t)$ is described as

$$X_i(t+1) \equiv \frac{R_i(t+1)}{R_i(t)} = a_i, \tag{1}$$