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Reliability of the optimized perturbation theory in the 0-dimensional $O(N)$ scalar field model

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HIGHLIGHTS

- The main advantages of the OPT method are its easy implementation.
- PMS used in conjunction with the OPT method tends to always produce better results.
- Our results motivate the use of the OPT method as a powerful nonperturbative technique.

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ABSTRACT

We address the reliability of the Optimized Perturbation Theory (OPT) in the context of the 0-dimensional $O(N)$ scalar field model. The effective potential, the self-energy and the 1PI four-point Green's function for the model are computed using different optimization schemes and the results contrasted to the exact results for the model. Our results are also compared to those obtained with the $1/N$-expansion and with those from ordinary perturbation theory. The OPT results are shown to be stable even at large couplings and to have better convergence properties than the ones produced in the $1/N$-expansion. It is also shown that the principle of minimal sensitive optimization procedure used in conjunction with the OPT method tends to always produce better results, in particular when applied directly to the self-energy.

1. Introduction

Perturbation theory is the most comprehensive way of studying nonlinear problems in physics area wide. However, it is a fact that not always we can rely on some small quantity or parameter in the model to generate a perturbative series. Even when a small parameter is available, it is not warranted that a perturbative series might be well posed, or converge after a few terms in the expansion are considered. Most of the times we must make use of some nonperturbative method to get around these problems. One typical example where perturbation theory breaks down is in the studies of phase transitions in general, particularly close to a critical point. This can also happen due to the appearance of large infrared divergences [1], as in the case where massless modes are present, or close to a transition point in field theories displaying a second order phase transition [2] or a weakly first order transition [3]. In all these cases, the use of some reliable nonperturbative technique

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is required to properly study these systems. Among the analytical nonperturbative techniques, one can cite for example making use of a discretization of the system and studying it numerically (e.g., lattice simulations), make use of analytical methods like an expansion in the number of field components, N, in the case of field theory, using the $1/N$-approximation [4], among other methods.

In this work we want to access the reliability of one of those nonperturbative methods that have been used with some frequency in the literature: The optimized perturbation theory (OPT). The OPT is an analytical technique which combines the computational advantages of ordinary perturbation theory to a variational criterion in order to generate nonperturbative results [5]. The OPT method has been used extensively in the literature to treat many different physical systems, ranging from condensed matter problems, phase transition problems in finite temperature quantum field theory and others (see, e.g., Refs. [6–22] and references therein for some examples of applications).

When applying the OPT method to a gauge theory, a modified form of the method is required. In this case, a suitable modification preserving gauge invariance can be implemented. This modification of the OPT method is known in this case as the Hard Thermal Loop perturbation theory (HTLpt) (see, e.g., the original proposal in Ref. [23] and also the review [24]).

One particular issue regarding the OPT method that we would like to also address in this work regards the quantity to which one should apply the variational criterion as required by the method. In a calculation where different physical quantities are available, in the original proposal by Stevenson [25], the variational principle used was the principle of minimal sensitivity (PMS) and it was advocated that the PMS should be applied to each different physical quantity that is being computed, producing different optimized parameters. However, one could argue that the PMS should be applied to a more general quantity such as the ground-state energy density as in Ref. [26], or to the effective potential, which generates all one-particle irreducible contributions, as in Refs. [11,13–15,27], while previous works [28,29] have shown that applying the PMS to the self-energy would be more appropriate. In this work we want to clarify in this issue which quantity we should optimize in the OPT method and also which quantity can provide the best convergence in the OPT. With this aim we shall compare the results obtained by a direct optimization of the effective potential (the zero-point Green function), the self-energy (the two-point Green function) and also to the effective coupling (the four-point Green function) in the context of the 0-dimensional $O(N)$ scalar field theory model.

One should note that one of the main differences as far as a comparison to a quantum field theory model in $D > 0$ is concerned, is the need to regularize and renormalize physical quantities, which is, of course, absent in the zero-dimensional model studied here. The renormalization group flow dictates the change of physical parameters with the scale and in this case the application of the OPT has to be handled very carefully [30–32]. Note, however, that the OPT method is not restricted to renormalizable models and it has been applied successfully to many effective nonrenormalizable models as well [26,33, 34]. Even so, the application of the OPT to the present exact soluble model offers a unique opportunity to elucidate on the possible optimization criteria issues, which are not possible to perform in other models without exact solutions. Because of this, the 0-dimensional $O(N)$ scalar field theory model is the perfect benchmark toy model to use to perform different tests related to the application of the OPT method, but it is also useful to test other different nonperturbative methods used in quantum field theory as well.

The remainder of this work is organized as follows. In Section 2, we briefly describe the 0-dimensional $O(N)$ scalar field model and show why perturbation theory might not be reliable in the context of this model in special. In Section 3, we introduce the OPT method and also describe three main variational tools that are commonly used in conjunction with this method. In Section 4, we perform a comparison between exact results and the nonperturbative results obtained by OPT. These results are also contrasted with those obtained from the $1/N$-expansion. This way we can better evaluate the usefulness of the OPT and the corresponding variational methods with this popular nonperturbative method. Our concluding remarks are given in Section 5. Four appendices are included where we give some of the technical details.

2. The 0-dimensional $O(N)$ scalar field model

The 0-dimensional $O(N)$ scalar field model describes an N-component anharmonic oscillator in zero spacetime dimension, whose action is given by

$$S(\boldsymbol{\varphi}) = \frac{m}{2}\boldsymbol{\varphi}.\boldsymbol{\varphi} + \frac{\lambda}{4!}(\boldsymbol{\varphi}.\boldsymbol{\varphi})^2, \tag{1}$$

where m, λ are real and positive parameters and $\boldsymbol{\varphi} \equiv (\varphi_1, \ldots, \varphi_N)$ is a scalar field with N components. Eq. (1) is an invariant under $O(N)$ rotations.

The generating function for the n-point Green's functions is given by

$$Z(\boldsymbol{J}) = \int D\boldsymbol{\varphi} \exp\left[-S(\boldsymbol{\varphi}) + \boldsymbol{J}\cdot\boldsymbol{\varphi}\right], \tag{2}$$

where $\boldsymbol{J}$ is an external source. From the generating function, the n-point Green functions are given by

$$G^{(n)}_{i_1\cdots i_n} = \frac{1}{Z}\frac{\delta^n Z(\boldsymbol{J})}{\delta J_{i_1}\cdots\delta J_{i_n}}\bigg|_{J_i=0} = \langle\varphi_{i_1}\cdots\varphi_{i_n}\rangle, \tag{3}$$