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Time series characterization via horizontal visibility graph and Information Theory

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HIGHLIGHTS

- This work deals with the characterization of dynamical systems using Horizontal Visibility Graphs (HVG) and Information Theory quantifiers.
- We propose the use of the weight distribution, which is based on the difference of the time series values of connected points.
- We study fractional Brownian motion time series and a paleoclimatic proxy record of ENSO taken from Pallcacocha Lake.
- The weight distribution allows a better characterization of the studied systems, using considerable shorter time series.

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ABSTRACT

Complex networks theory have gained wider applicability since methods for transformation of time series to networks were proposed and successfully tested. In the last few years, horizontal visibility graph has become a popular method due to its simplicity and good results when applied to natural and artificially generated data. In this work, we explore different ways of extracting information from the network constructed from the horizontal visibility graph and evaluated by Information Theory quantifiers. Most works use the degree distribution of the network, however, we found alternative probability distributions, more efficient than the degree distribution in characterizing dynamical systems. In particular, we find that, when using distributions based on distances and amplitude values, significant shorter time series are required. We analyze fractional Brownian motion time series, and a paleoclimatic proxy record of ENSO from the Pallcacocha Lake to study dynamical changes during the Holocene.

1. Introduction

In the last few years, methods to transform time series into networks have been proposed, and with them, novel ways to analyze and characterize time series, have been developed. Among others, these novel methodologies include the use of disjoint cycles and their distances in the phase space to generate the links in the corresponding network [1,2]. Li and Wang

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[3,4] introduce a method based on n-tuples. Donner et al. [5,6] work with recurrence networks. There are also methods based on the phase space reconstruction of the time series [2,7,8]. Latora et al. [9] propose a graph based on the recurrence of time series motifs. Other methods take into account the visibility of elements in a time series, like the Visibility Graphs or the Horizontal Visibility Graphs [10,11]. Our article focuses on the use of the latter.

Following previous works [12,13], we extract probability distribution functions (PDFs) from the constructed networks to characterize the topological structure and to capture the dynamics of the transformed time series, using Information Theory quantifiers. Related works have primarily focused on the network's degree distribution. We investigate in this work, alternative probability distributions and we compare their performance with the usual degree distribution. Specifically, we explore the distance distribution, that despite being poorly explored, it was shown to be efficient in capturing network's topological changes [14]. We also propose a PDF based on the difference of the time series values (amplitudes) between the nodes connected by the horizontal visibility algorithm. We find the distance distribution and the one based on amplitude differences more efficient in characterizing the studied systems as they require significantly shorter time series than the degree distribution.

We study fractional Brownian motion (fBm) time series generated with different degrees of correlations (different Hurst exponents), and a paleoclimatic proxy record of the Laguna Pallcacocha used to study the millennial El Niño/Southern Oscillation (ENSO) dynamic.

2. Horizontal visibility graph and associated PDFs

The horizontal visibility graph (HVG) is a methodology that transforms a time series into a graph maintaining the inherent characteristics of the transformed time series [11]. The HVG consists in a geometrical simplification of the firstly proposed visibility graph (VG) [10]. It considers each point in the time series, a node in the network, connected by the following consideration: Let $\{x_i,\ i = 1, \ldots, N\}$, be a time series of N data. Two nodes i and j in the graph are connected if it is possible to trace a horizontal line, in the time series, linking x_i and x_j not intersecting intermediate data height, fulfilling: $x_i,\ x_j > x_n$ for all $i < n < j$.

In the HVG, the nodes can see at least its nearest neighbors, incorporating in a natural way the time causality. One of the properties of the HVG is that it is not modified under rescaling of horizontal and vertical axes, as well as under horizontal and vertical translations [11,15].

2.1. Probability distributions extracted from HVG

Once the graph is constructed, several ways of extracting information about its structure are possible. The most usual one is to extract the degree distribution that describes the way the node's degrees are distributed in the graph. The degree distribution has been used to study several natural and artificial systems, from river flows [16], to laser intensity analysis [17]. For a given network G with N nodes, the degree distribution, $P_{deg}(\kappa)$, is the fraction of nodes with degree κ. This discrete distribution is defined on the set $\{0, 1, \ldots, N-1\}$.

Other probability distributions, such as the distance distribution have been still poorly explored, however, one recent work has shown the distance distribution to be very effective in capturing network's topological changes [14]. The distance between a pair of nodes is the shortest path between them, thus, the distance distribution, $P_\delta(d)$, is the fraction of pairs of nodes at distance d. The maximum possible distance is $N-1$, and when a pair is disconnected, we consider ∞, thus, the distance distribution is discrete and defined over the set $\{1, 2, \ldots, N-1, \infty\}$.

In this article, we also explore a straightforward modification of HVG, that consists in weighting the edges based on the difference between two connected values in the time series. The weight of an edge is a real value proportional to the amplitude difference between two connected points. Considering $\mathbf{x} = \{x_1, \ldots, x_n\}$ a sample of n real values, if x_i and x_j are connected, the edge (i, j) has a weight $w_{ij} = x_i - x_j$. It is important noticing that, if we keep track of the first value of the time series, we could reconstruct the series from the resulting graph. As w_{ij} is a continuous variable, a histogram is constructed to estimate the probability distribution. $P_w(A)$ represents the fraction of edges with amplitude A. Fig. 1 exemplifies how these PDFs are obtained from a time series.

3. Information theory quantifiers

3.1. Shannon entropy

When considering discrete probability distributions ($P = \{p_j : j = 1, \ldots, M\}$) the *Shannon entropy* $S[P]$ [18] is defined as:

$$S[P] = -\sum_{j=1}^{M} p_j \cdot \ln p_j. \tag{1}$$

If $S[P] = 0$ we are in a position to predict with certainty *which* of the possible outcomes j whose probabilities are given by p_j will actually take place. Our knowledge of the underlying process described by the probability distribution is, in this