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Numerical study of free-fall arches in hopper flows

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HIGHLIGHTS

- DEM simulation of hopper flows were performed on GPUs.
- The free-fall arch structures in hopper flows were plotted by statistical methods.
- Spatial profiles in the hopper flows were investigated statistically.
- The arch structure is consistent with the profiles but different from ideal FFAs.

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ABSTRACT

Beverloo's law describes the flow rate of grains discharging from hoppers, where the assumption of a free-fall arch (FFA) is very useful in understanding the physical picture of this process. The FFA has been observed in previous experiments but a clear systematic study of the FFA is still necessary. In this paper, dense granular flow in hoppers was studied by numerical simulations, in attempts to explore the free-fall region and its boundary. Generally, the numerical simulation results support the free-fall arch assumption, although the statistical description of the FFA is not exactly equivalent to its strict definition.

1. Introduction

The discharge of granular materials from hoppers has been widely studied for decades [1–7]. Unlike normal fluids, granular materials discharge constantly by gravity independent of filling height. Furthermore, the discharge rate is independent of the hopper width and height when these dimensions are more than 2.5 times of the outlet diameter [2]. The widely accepted formula of flow rates called 'Beverloo's law' was proposed by Beverloo et al. in 1961 [1]. This formula has a unified form ($\phi = C\rho\sqrt{gR}A$) for various types of hoppers, where C is a dimensionless coefficient, R is the equivalent hydrodynamic radius, ρ is the equivalent density, g is the acceleration of gravity and A is the equivalent hydrodynamic section. The formula is valid when R is much larger than the particle's size [4,8]. For a two dimensional hopper in particular, $R = 0.5(D_0 - kd)$ and $A = D_0 - kd$ where D_0 is the width of the outlet and d is the particle diameter. In this case the law can be rewritten as $\phi = C_0\rho W\sqrt{g}\,(D_0 - kd)^{1.5}$ [2,9], where W is the thickness of the hopper. Similarly for three-dimensional hoppers with round outlets, the formula is $\phi = C_0\rho\sqrt{g}\,(D_0 - kd)^{2.5}$ [1] and for hoppers with slot outlets, it is $\phi = C_0\rho\sqrt{g}\,(L_x - kd)\left(L_y - kd\right)^{3/2}$ [10,11], where L_x is the length of the long side of the slot and L_y is the length of the short side.

An arch structure referred to as a free-fall arch (FFA) was assumed to exist above the outlet to understand the law [6,7,9,12]. In this assumption, the velocities of grains above the arch are negligible and grains fall freely due to gravity below the arch. If the height of the FFA is proportional to the outlet diameter, the grains' velocities at the outlet should be

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proportional to $\sqrt{gR}$. With A being the cross-sectional area that the grains flow through, the volume flowed out per unit time should be $\sqrt{gR}A$, and the discharge rate should be proportional to $\rho\sqrt{gR}A$, where ρ is the equivalent density.

As noted above, the assumption of FFA in funnel flow provides a good explanation for Beverloo's law. The concept of FFA was firstly introduced by Hagen in the mid nineteenth century [13,14]. In 1961, Brown developed a 'minimum energy theorem' to describe granular flow that falls freely under gravity near apertures [15], and later he and Richards photographed the free-fall region above the outlet in a two-dimensional rectangular frame in hopper flows of ball bearings. They named the boundary of this region the 'free-fall arch' [16,17]. Le Pennec et al. observed the FFA in flowing hour-glasses when the flow was similarly intermittent [18]. Barletta et al. generalized the model to two-phase flow of gas and grains [19]. Furthermore, the FFA theory was applied to explain experiments in various situations, such as funnel flows and mass flows in conical hoppers [20] and tilted hoppers [21]. In recent years, the FFA has also been considered in several computer simulations of granular flows that use various calculation methods [5,6,12,22,23]. Recently Vivanco's experiment in wedge hoppers using photo elastic plates suggested that there were no permanent free-fall arches during the flow, but a time-averaged of the network of contact forces could show a boundary with characteristics resembling the FFAs when the outlet was not large [24]. Because the characteristics of FFAs are difficult to study by direct observation in experiments, the detailed physical picture of FFA is still unclear [25], and the relationship between the FFA and parameters in various granular flows is obscure.

In this work, dense granular flows were studied in two-dimensional flat-bottomed hoppers by numerical simulations. Spatial profiles of velocities, the ratio of spheres with the acceleration of gravity to all the spheres in a unit area and the contact number in the funnel flows near the outlet were investigated statistically. Then a statistical description of the shapes of the arch structures was developed by analyzing the simulation results for these physical quantities. At last, the free fall region was tested by considering velocities and the flow rates were cross-checked.

2. Methods

2.1. The simulation model

In simulations of granular materials, the discrete element method (DEM) [26] has been widely accepted as an effective method to solve problems regarding granular materials including granular flows [27–30]. To carry out a simulation using the DEM, two steps are required: (1) calculation of forces among the grains; (2) time integration of transitional and rotational velocities and accelerations. In this work, the grains were assumed to be spheres and the Hertz–Mindlin contact model [3,26] was applied to calculate forces among these spheres. Quantities such as sphere diameter d (radius $r = d/2$), mass m, moment of inertia I, elastic modulus E, shear modulus G, Poisson's ratio ν, friction coefficient μ and coefficient of restitution ε are involved in this model. In particular, if two identical contacting spheres i and j are in positions $\mathbf{r}_i$ and $\mathbf{r}_j$, with velocities $\mathbf{v}_i$ and $\mathbf{v}_j$, the contact force between them is composed of the normal force $\mathbf{F}_{ijn}$ and the tangential force $\mathbf{F}_{ijt}$, as follows:

$$\begin{cases} \mathbf{F}_{ijn} = k_n\boldsymbol{\delta}_{ijn} - \gamma_n\mathbf{v}_{ijn}, \\ k_n = \dfrac{4}{3}E\sqrt{2r\delta_{ijn}}, \\ \gamma_n = -2\sqrt{\dfrac{5}{6}}\beta\sqrt{2E\sqrt{\dfrac{r}{2}\delta_{ijn}}m}, \end{cases} \tag{1}$$

$$\begin{cases} \mathbf{F}_{ijt} = k_n\boldsymbol{\delta}_{ijt} - \gamma_n\mathbf{v}_{ijt}, \\ k_t = 4G\sqrt{2r\delta_{ijn}}, \\ \gamma_t = -2\sqrt{\dfrac{5}{6}}\beta\sqrt{2G\sqrt{\dfrac{r}{2}\delta_{ijn}}m} \end{cases} \tag{2}$$

where $\boldsymbol{\delta}_{ijn}$ and $\boldsymbol{\delta}_{ijt}$ are the normal and tangential displacement vectors and δ_{ijn} and δ_{ijt} are their modules, respectively; $\mathbf{v}_{ijn}$ and $\mathbf{v}_{ijt}$ are the normal and tangential relative velocities between spheres i and j [26]; k_n and k_t are the normal and tangential elastic coefficients; γ_n and γ_t are the normal and tangential damping coefficients. The total force exerted between the spheres is

$$\mathbf{F}_{ij} = \mathbf{F}_{ijn} + \mathbf{F}_{ijt}. \tag{3}$$

Moreover if there is friction, the Coulomb yield criterion $|\mathbf{F}_{ijt}| \leqslant |\mathbf{F}_{ijn}|$ is satisfied by truncating the magnitude of $\mathbf{F}_{ijt}$. As a result, if $|\mathbf{F}_{ijt}| > |\mu\mathbf{F}_{ijn}|$, $|\mathbf{F}_{ijt}| = |\mathbf{F}_{ijn}|\mathbf{u}_{ij}/|\mathbf{u}_{ij}|$.

Hence, considering these forces in the gravity field, the equations of motion of the ith sphere are:

$$\begin{cases} m_i\mathbf{a}_i = \sum_j \mathbf{F}_{ij} + m_i\mathbf{g}, \\ I_i\boldsymbol{\beta}_i = -\sum_j \dfrac{r_i}{r_{ij}}\mathbf{r}_{ij} \times \mathbf{F}_{ij} \end{cases} \tag{4}$$

where $\mathbf{a}_i$ is the acceleration of the ith sphere and $\boldsymbol{\beta}_i$ the angular acceleration. As Eqs. (1)–(3) are combined together to describe the motion of spheres, the Velocity-Verlet scheme [31] was applied to perform time integration to numerically solve Eq. (4). Then, every single sphere was simulated by considering the Hertz model.