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Superstatistical analysis of sea-level fluctuations

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HIGHLIGHTS

- First application of superstatistics to analyse sea-level fluctuations.
- We perform a careful comparison of statistical differences between various locations.
- Time scale separation necessary for superstatistics approach verified for sea-level data.

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ABSTRACT

We perform a statistical analysis of measured time series of sea levels at various coastal locations in the UK, measured at time differences of 15 min over the past 20 years. When the astronomical tide and other deterministic components are removed from the record, a stochastic signal corresponding to the meteorological component remains, and this is well-described by a superstatistical model. We do various tests on the measured time series, and compare the data at 5 different UK locations. Overall the χ^2-superstatistics is best suitable to describe the data, in particular when one looks at the dynamics of sea-level *differences* on short time scales.

1. Introduction

Many complex nonequilibrium phenomena can be understood as the superposition of different dynamics on different time scales. The superstatistical approach [1] models these complex phenomena as the superposition of two random variables, one that corresponds to local equilibrium statistical mechanics and another one that corresponds to a slowly varying parameter β, which for example can be the local inverse temperature in a spatio-temporal inhomogeneous system, or some local variance parameter associated with slices of a given size in a given time series. In nonequilibrium statistical mechanics, this technique is a powerful tool to describe a large variety of complex systems for which there is a spatial or temporal change of conditions on a large scale, larger than the local relaxation time [2–10]. Effectively, when integrating out the fluctuations of the parameter, this leads to more general types of statistical mechanics, described by more general entropy functions [10–12]. Essential for superstatistical models is sufficient time scale separation, i.e. the local relaxation time of the system must be much shorter than the typical time scale on which the parameter β changes. Many interesting applications of the superstatistics concept have been worked out for a variety of complex systems, for example the analysis of train delay statistics [13], hydrodynamic turbulence [14], cancer survival statistics [15] and much more [16–21]. Typical distributions of the superstatistical parameter β that occur in many of these applications are the χ^2, inverse χ^2 or lognormal distribution [2]. Superstatistics based on χ^2 distributions leads to q-statistics [11,22], whereas other distributions lead to more complicated versions of generalized statistical mechanics [10].

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Of particular interest are superstatistical techniques to analyse and model the complexity inherent in environmental time series, such as rainfall, wind, surface temperature, and related quantities. Rapisarda et al. [17], Kantz et al. [23] and Stoevesandt et al. [41] investigated superstatistical aspects of wind velocity fluctuations. Yalcin et al. [24] did a data analysis of relevant surface temperature distributions on the earth, which is important if one wants to understand the effective statistical mechanics for thermodynamic devices (or local ecosystems) that are kept in the open air outside a constant-temperature environment. Porporato et al. [25] looked at rainfall statistics and developed a model where the rate parameter of the underlying Poisson process fluctuates in a superstatistical way.

A central point for the applicability of the superstatistics concept is the existence of suitable time scale separation of the dynamical evolution, or more generally the existence of a hierarchy of time scales which are well separated. In the simplest case this just means there are two different time scales such that the typical variation of β takes place on a much larger time scale than the local relaxation time of the system that is influenced by the given temperature environment. There are tests that can check for a given experimental time series if such a time scale separation is present [8].

In this paper we will deal with a new application of the superstatistics concept that has not been considered before, namely the superstatistical analysis of measured sea-level fluctuations at various spatial locations in the UK. Of particular interest are: (a) which type of superstatistics is relevant for sea levels; (b) how the results differ from spatial location to location; (c) how well the time scale separation criterium is satisfied and (d) what the relevant time scales are. Our data set consists of a record of measured sea levels at various tide gauge locations in the UK that were measured every 15 min over the past 20 years or so. In Section 2 we review the derivation and application of the superstatistical approach and the different tools to check its suitability for a particular data set, in our case sea-level data. In Section 3 we analyse the data set of observed sea levels at five different sites in the UK, and provide answers to questions (a)–(d). Our concluding remarks are given in Section 4.

2. Superstatistical analysis of time series

2.1. The superstatistical model

Consider a stochastic process that for a short time frame is well described by a Gaussian distribution, but on a longer time scale the parameter value β of this Gaussian fluctuates. Concretely, for a given value of β the conditional probability distribution is given by

$$p(v \mid \beta) \sim \exp\left(-\frac{\beta v^2}{2}\right).$$

Assume that there exists a probability distribution $f(\beta)$ describing the fluctuations of β, then the density function is expected to behave as

$$p(v) = \int_0^\infty f(\beta)p(v \mid \beta)\mathrm{d}\beta \sim \int_0^\infty f(\beta)\mathrm{e}^{-\beta v^2/2}\mathrm{d}\beta. \tag{1}$$

There are three different superstatistics distributions which are commonly found in many applications:

χ^2-superstatistics, also known as Tsallis statistics. The function $f(\beta)$ is given by the Γ-distribution

$$f(\beta) = \frac{1}{\Gamma\left(\frac{n}{2}\right)}\left(\frac{n}{2\beta_0}\right)^{n/2}\beta^{n/2-1}\mathrm{e}^{n\beta/2\beta_0},$$

where β_0 is the average of β and n is the number of degrees of freedom.

Inverse χ^2-superstatistics. In this case $f(\beta)$ is given by the inverse Γ-distribution

$$f(\beta) = \frac{\beta_0}{\Gamma\left(\frac{n}{2}\right)}\left(\frac{n\beta_0}{2}\right)^{n/2}\beta^{-n/2-2}\mathrm{e}^{n\beta_0/2\beta},$$

where β_0 is again the average of β and n is the number of degrees of freedom of the inverse χ^2 distribution.

Lognormal superstatistics. In this last case $f(\beta)$ is described by the lognormal distribution

$$f(\beta) = \frac{1}{\sqrt{2\pi}s\beta}\exp\left(\frac{-\left(\ln\frac{\beta}{\mu}\right)^2}{2s^2}\right),$$

where μ and s are suitable parameters.

2.2. Tools for data analysis

Consider $u = \{u_1, \ldots, u_n\}$ as a set of n experimental observations (or data points in general). The total time series is divided into N slices of length Δ, with $N = \lfloor n/\Delta \rfloor$ where $\lfloor\cdot\rfloor$ denotes the integer part (or floor) function. The local moment