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Scattering in thick multifractal clouds, Part II: Multiple scattering

S. Lovejoy [a,*], B.P. Watson [b], Y. Grosdidier [a], D. Schertzer [c]

[a] Department of Physics, McGill University, 3600 University Street, Montreal, Que. H3A 2T8, Canada

[b] Department of Physics, St. Lawrence University, Canton, NY 13617, USA

[c] CEREVE, Ecole Nationale des Ponts et Chaussées, 6-8, avenue Blaise Pascal, Cité Descartes, 77455 MARNE-LA-VALLEE Cedex, France

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ABSTRACT

In Part I of this paper, we developed asymptotic approximations for single photon scattering in thick, highly heterogeneous, "Log-Lévy" multifractal clouds. In Part II, theoretical multiple scattering predictions are numerically tested using Monte Carlo techniques, which show that, due to long range correlations, the photon paths are "subdiffusive" with the corresponding fractal dimensions tending to increase slowly with mean optical thickness. We develop reasonably accurate statistical relations between *N* scatter statistics in thick clouds and single scatter statistics in thin clouds. This is explored further using discrete angle radiative transfer (DART) approach in which the radiances decouple into non-interacting families with only four (for 2-D clouds) radiance directions each. Sparse matrix techniques allow for rapid and extremely accurate solutions for the transfer; the accuracy is only limited by the spatial discretization.

By "renormalizing" the cloud density, we relate the mean transmission statistics to those of an equivalent homogeneous cloud. This simple idea is remarkably effective because two complicating effects act in contrary directions: the "holes" which lead to increased single scatter transmission and the tendency for multiply scattered photons to become "trapped" in optically dense regions, thus decreasing the overall transmission.

1. Multiple scattering and non-conservative multifractals

From the point of view of applications, the single scattering results of Part I have severe limitations. First, it is not obvious how those calculations are relevant to multiple scattered radiation in thick clouds. In addition, even for single scattering, it is not clear that the results will be relevant in non-conservative clouds in which the cloud density is related to the (scale by scale) conservative multifractal fluxes by a fractional integration as outlined in Section 2.1 of Part I. We mentioned that the Corrsin–Obukov law for passive scalar advection yields $H = 1/3$ and $\varphi = \chi^{\frac{1}{2}}\varepsilon^{-\frac{1}{6}}$ where χ is the passive scalar variance flux and ε is the energy flux. Observations of aerosols [1–4] and cloud liquid water [5,6] show that in the horizontal H is indeed close to 1/3. Unfortunately, the analytic treatment of the statistics of the fractionally integrated flux (FIF) model with $H \neq 0$ is too difficult; the same is true of extending the above directly to multiple scattering since successive scatters are strongly correlated. We will therefore turn to numerical simulations.

Fig. 1a and b show 12 multifractal simulations on 512^2grids of the conserved flux φ (i.e. $H = 0$) and their fractional integration by $H = 1/3$ respectively, showing the large realization-to-realization variability as well as the smoothing

* Corresponding author.
E-mail address: lovejoy@physics.mcgill.ca (S. Lovejoy).

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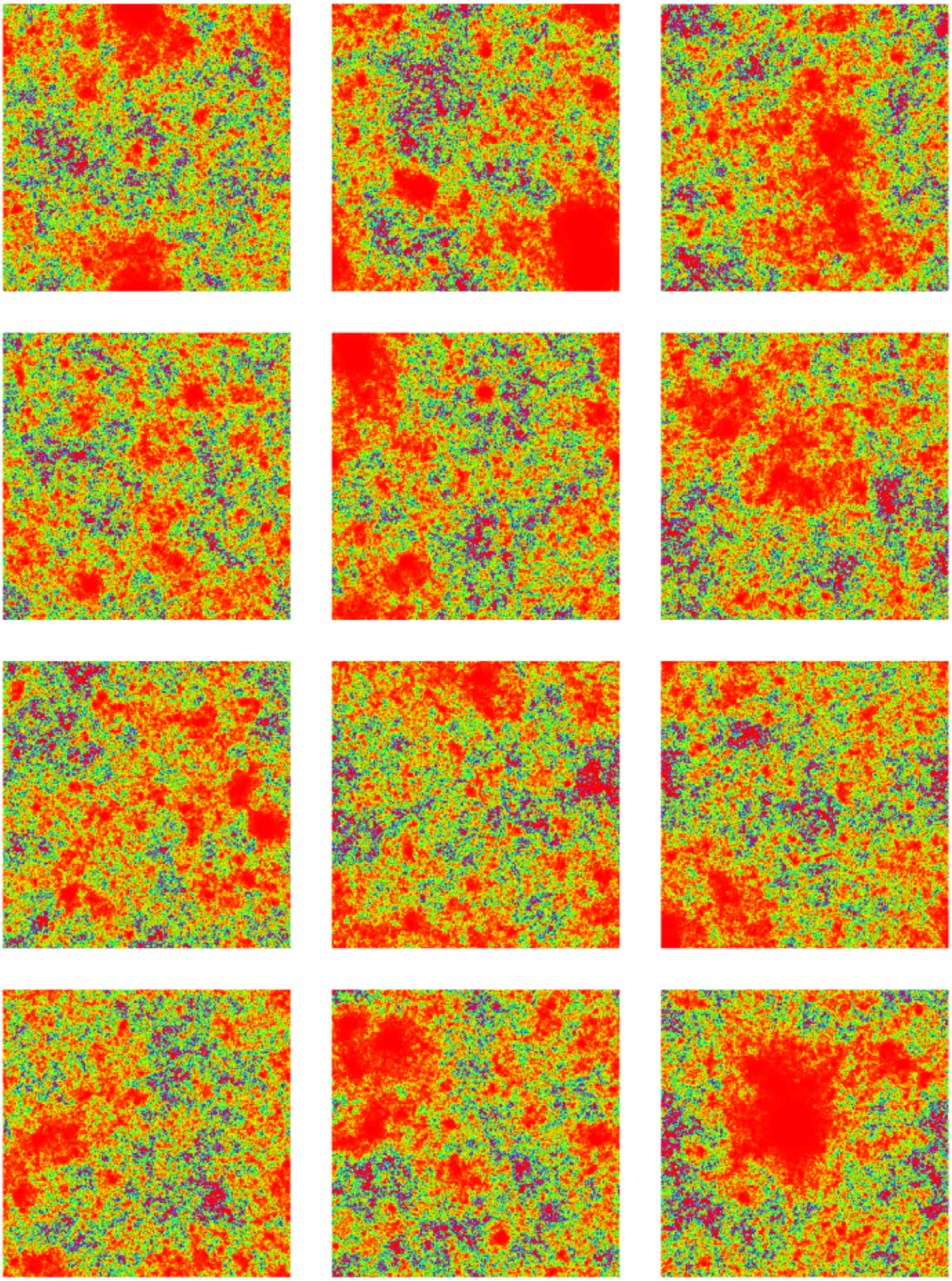

Fig. 1. (a, b) The twelve cloud density fields on 512 × 512 grids used for the Monte Carlo calculations. Above: $H = 0$. Red–orange indicates low density, purple indicates high density. Below are the same fields fractionally integrated with $H = 1/3$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

introduced by H. In both cases, ensemble mean of the spatially averaged density ρ is unity but the standard deviations of the realization by realization spatial means are 0.56 and 0.55 for $H = 0$, $H = 1/3$ respectively.