Coherence resonance induced by non-Gaussian noise in a deterministic Hodgkin–Huxley neuron☆

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ABSTRACT

In this paper, we report a novel effect of a particular kind of non-Gaussian noise (NGN), especially its deviation from Gaussian noise, on the spiking activity in a deterministic Hodgkin–Huxley (HH) neuron. It is found via numerical simulations that the coefficient of variation (CV), characterizing the spiking regularity, nonlinearly changes with varying deviation and passes through a minimum at an optimal deviation value, representing the presence of the coherence resonance (CR). This phenomenon shows that, when the NGN is optimal, the neuron can exhibit the most regular firing behavior. This result provides a constructive role of the NGN in the timing precision of information processing and signal transduction in neural systems.

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1. Introduction

Since the study of neuronal spiking dynamics by Hodgkin and Huxley (HH) [1], numerous studies have contributed to the effect of noise on the generation and propagation of the membrane potential in both deterministic and stochastic HH neurons [2–31]. Noise in a neuron can be sorted into two types: External noise, mainly coming from synapses, and internal noise, coming from stochastic ion channels. Stochastic resonance (SR) and coherence resonance (CR) due to external noise in the deterministic HH model and related models of excitable dynamics have extensively been studied [2–9]. Due to the more importance of channel noise, there have been more works contributed to the stochastic HH neuron firing [10–31]. Channel noise originates from the stochastic nature of the ion channel dynamics, specifically, the origin of the channel noise is basically due to the fluctuations of the mean number of open ion channels around the corresponding mean values [10–13]. Studies show that channel noise has features such as the threshold to spiking and the spiking rate itself [14–20], the anomalous noise assisted enhancement of transduction of external signals [21–31], and the efficiency for synchronization [28–31].

In the previous works, both external and internal noises are usually assumed to be of Gaussian behaviors. However, experimental results for one kind of crayfish [3] and for rat skin [4] offer strong indications that there could be non-Gaussian colored noise sources in these sensory systems. Over the last decade, a particular kind of non-Gaussian noise (NGN) has been widely studied in various nonlinear dynamical systems of physics, chemistry, and especially of biology [32–42] like the so-called genetic model that follows from a particular biological modeling [37,43]. This proves that it is reasonable to study the NGN on the dynamics of excitable systems like the HH neuron that exhibits the spiking form of "all-or-nothing". On the other hand, the non-Gaussianity of the NGN is characterized simply by a parameter q that measures the deviation from Gaussian noise ($q = 1$) and determines the probability distribution of the noise. Thus, such a particular noise form allows one to easily control the deviation from Gaussian behavior by changing a single parameter q.

In this paper, we apply the NGN to a deterministic HH neuron and numerically study the effect of the NGN, specifically, the deviation from Gaussian noise and the deviation-dependent noise intensity and correlation time, on the firing behavior in the neuron. We find a novel phenomenon that the coefficient of variation (CV), which characterizes the spiking regularity, nonlinearly changes with increasing deviation and passes through a minimum at an optimal deviation value, which represents the presence of the CR. This phenomenon shows that when the NGN, including the deviation and the deviation-dependent noise intensity and correlation time, is optimal, the neuron can reach most regular firing behaviors.

2. Model and equations

According to the HH model, the ion current across the biological membrane is carried mainly by the motion of sodium (Na^+) and potassium (K^+) ions through selective and voltage-gated ion channels embedded across the membrane. In addition, there is a leakage current present that is associated with other ions. The gating dynamics of each ion channel is governed by four independent gates, each of which can switch between an open and a closed conformation. Consequently, in the absence of an external stimulus, the dynamics of the membrane potential of the HH neuron involving external synaptic noise is given by

$$C\frac{dV}{dt} = -g_{Na}^{max}m^3h(V-V_{Na}) - g_K^{max}n^4(V-V_K) - G_L(V-V_L) + \xi(t) \tag{1a}$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)m, \tag{1b}$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)h, \tag{1c}$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)n, \tag{1d}$$

with voltage-dependent opening and closing rates:

$$\alpha_m(V) = \frac{0.1(V+40)}{1-\exp[-(V+40)/10]}, \tag{2a}$$

$$\beta_m(V) = 4\exp[-(V+65)/18], \tag{2b}$$

$$\alpha_h(V) = 0.07\exp[-(V+65)/20], \tag{2c}$$

$$\beta_h(V) = \{1+\exp[-(V+35)/10]\}^{-1}, \tag{2d}$$

$$\alpha_n(V) = \frac{0.01(V+55)}{1-\exp[-(V+55)/10]}, \tag{2e}$$

$$\beta_n(V) = 0.125\exp[-(V+65)/80]. \tag{2f}$$

Here, V denotes the deviation of the membrane potential of the neuron from its equilibrium state, and $C = 1\,\mu F\,cm^{-2}$ is the membrane capacity. $g_{Na}^{max} = 120\,ms\,cm^{-2}$ and $g_K^{max} = 36\,ms\,cm^{-2}$ are the maximal sodium and potassium conductances, respectively. $V_{Na} = 50\,mV, V_K = -77\,mV$, and $V_L = -54.4\,mV$ are the reversal potentials for the sodium, potassium, and leakage currents. The leakage conductance here is assumed to be constant, $G_L = 0.3\,ms\,cm^{-2}$. The gating variables m, h, and n describe the mean ratios of the open gates of the working channels [18], and the factors n^4 and m^3h are the mean portions of the open ion channels within the membrane patch.

The NGN $\xi(t)$ has a particular non-Gaussian distribution form as [32–42]

$$\frac{d\xi(t)}{dt} = -\frac{1}{\tau}\frac{d}{d\xi}V_q(\xi) + \frac{\sqrt{2D}}{\tau}\Gamma(t) \tag{3a}$$

and

$$V_q(\xi) = \frac{D}{\tau(q-1)}\ln\left[1+\frac{\tau}{D}(q-1)\frac{\xi^2}{2}\right], \tag{3b}$$

where $\Gamma(t)$ is the Gaussian white noise with zero mean and auto-correlation function $\langle\Gamma(t)\Gamma(t')\rangle = \delta(t-t')$, D and τ are the parameters related to the noise intensity and the correlation time, respectively, and q is the deviation of the NGN from Gaussian behavior. The stationary probability distribution has the form [36]:

$$P_q^{st}(\xi) = \frac{1}{Z_q}\left[1+\frac{\tau}{D}(q-1)\frac{\xi^2}{2}\right]^{-1/q-1}, \tag{4}$$

where Z_q is the normalization factor. This distribution can be normalized only for $q < 3$. The first moment $\langle\xi\rangle = 0$, is always equal to zero, and the second moment $\langle\xi^2\rangle = 2D/[\tau(5-3q)]$ is finite only for $q < 5/3$. Furthermore, for $q < 1$,