Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 368 (2006) 541–550
PHYSICA A
www.elsevier.com/locate/physa

# Controlling traffic flow near the transition to the synchronous flow phase

L.C. Davis

*Physics Department, University of Michigan, Ann Arbor, MI 48109, USA*

Received 22 July 2005; received in revised form 25 October 2005
Available online 17 January 2006

**Abstract**

For constant incoming flow far upstream of a freeway on-ramp, the flow downstream (throughput) and the rate of merging are studied with simulations using a generalized optimal velocity model. For large enough merge rates, a transition to synchronous flow occurs and the throughput is reduced by 0.5–0.7 vehicle on average for each vehicle that merges. For smaller merge rates there is free flow on the freeway and the throughput is the sum of the merge rate and the flow upstream of the on-ramp. Thus, there is an optimum merge rate that maximizes the throughput for a given incoming flow rate. These results hold for a wide range of initial vehicle position and velocity profiles and for single- as well as double-lane freeways. The results show that the transition to synchronous flow is due to the dynamics of the merge process, rather than to a limitation on the capacity of the downstream portion of the freeway. As a consequence, a new on-ramp metering algorithm, which controls the merge rate to prevent the transition to synchronous flow and concomitantly to maximize flow, has been developed.


*Keywords:* Traffic model; Traffic flow; Traffic metering; Congested flow

## 1. Introduction

An important recent advance in traffic science is the discovery of the (non-equilibrium) phase transition free flow to synchronous flow, which generally occurs at a freeway on-ramp or other bottleneck [1–6]. It differs from a traffic jam is several respects; the downstream edge is pinned at the bottleneck and the average velocity and flow within the congested region are higher than in a jam.

There have been several papers describing flow near an on-ramp. Lee et al. [7] studied the dynamical states of flow using a continuum model. Huang [8] discussed highway on-ramp control, also using a continuum model. Berg and Woods [9] described solitary waves produced in simulations using a car-following model. Pedersen and Ruhoff [10] showed how to include an on-ramp in the Nagel–Schreckenberg cellular automata model. Kuhne et al. [11] developed a probabilistic theory for the formation of congestion at on-ramps [12]. For general reviews of traffic flow see Refs. [13,14]. Refs. [7–11,13] are not based on the three-phase theory and have been strongly criticized by Kerner [14] Section 3.3. Further understanding of the synchronous flow

*E-mail address:* ldavis7@peoplepc.com.

0378-4371/$ - see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2005.12.026

transition is necessary if effective freeway traffic control measures, such as metering [15–17], are to be implemented.

Many modern metering systems are based on the ALINEA algorithm (Asservissement LINeaire d'Entree Autroutiere) [18,19]. It adjusts the on-ramp merge rate to achieve a desired, set density of vehicles downstream of the on-ramp. If the average velocity is constant at the downstream occupancy detector, such a system in effect controls the flow (the density–velocity product). So a metering system using the ALINEA algorithm essentially controls flow to the capacity of the freeway. However, as Kerner as pointed out [20], even if free flow prevails downstream the capacity is not a fixed quantity but depends upon the conditions upstream of the on-ramp.

Recently, Kerner has made a detailed comparison of ALINEA to his new algorithm called automatic on-ramp control of congested patterns (ANCONA) [21]. Kerner based his algorithm on measuring the average velocity of vehicles passing a sensor just upstream of the on-ramp. When the average velocity falls below a critical velocity, on-ramp flow is restricted to a lower rate by metering. Congestion is allowed to build up at the on-ramp on the main line, but is localized there, in an attempt to maximize throughput.

The purpose of the present work is to find a metering algorithm that prevents the synchronous flow phase transition, yet maximizes throughput. This is done by examining the transition to and characteristics of synchronous flow as a function of the rate of merging. The organization of the paper is as follows. In Section 2 simulations for a single-lane freeway with an on-ramp are presented. The results for a double-lane freeway are given in Section 3. A new algorithm to meter the rate of merging from the on-ramp is suggested and evaluated in Section 4. My conclusions are discussed in Section 5.

## 2. Single-lane freeway with on-ramp

Simulations were performed using a vehicle model that satisfies the basic postulate of the three-phase model: steady-state solutions, which correspond to synchronized flow, exist in a two-dimensional region of the flow-density plane [5]. The phases are free flow, synchronous flow, and wide moving jams [1]. The model includes driver reaction time, $t_d$, as well as physical limits on vehicle acceleration and deceleration and other improvements on the earlier modified optimal velocity model of Ref. [22]. The dynamical equation for the velocity of vehicle $n$ is of the form

$$\tau_n \frac{\mathrm{d}v_n(t)}{\mathrm{d}t} + v_n(t) = V(\Delta x_n(t - t_d), \Delta v_n(t - t_d), v_{n-1}(t - t_d), v_n(t)), \tag{1}$$

where $\Delta x_n$ is the distance to the vehicle $n-1$ in front of $n$ and $\Delta v_n$ is velocity difference. The mechanical time constant of vehicle $n$ is $\tau_n$. Full details are described in the Appendix and in Ref. [23]. This model is one of only a few three-phase models in the literature. One is that of Kerner and co-workers described in Ref. [21] and references therein. Another is that of Jiang and Wu [24], who have developed a cellular automata model based on the three-phase theory.

A diagram of an on-ramp to a freeway is shown in Fig. 1 along with an indication of where the flow quantities $q_1$ and $q_3$ pertain. I take $q_1$ to be the uniform, free flow far upstream of the on-ramp. If congestion occurs, the flow into the merge region can be different from $q_1$. Likewise, $q_3$ is the flow downstream of the on-ramp and I assume that free flow conditions apply here as well, that is, there is no congestion coming from a bottleneck further downstream. The freeway can have multiple lanes, although I initially restrict the simulations to a single lane for simplicity. The rate of merging is $m$ and can differ from the flow into the on-ramp if a queue develops.

The simulations for the single-lane freeway were done for ~600 vehicles and a time interval of 500 s. Unless noted otherwise, the initial profile of vehicles is given by a power-law distribution of headways (including vehicle length) $h > h_0$, $P(h) \sim 1/h^{\mu+1}$, with a fraction $1-p$ removed ($p$ is the occupied fraction), and velocities near the speed limit. The possible initial vehicle positions are given for each lane by $x_{k+1} = x_k - h_k$ with $x_0 = 0$, $k = 1, 2\ldots$ and $h_k$ is the $k$th headway randomly selected from the power-law distribution $P(h)$. The lead vehicle on the freeway travels at the speed limit. Merging is allowed in a region of length $d_{merge}$ with $x = 0$ as the downstream end of the merge. The throughput $q_3$ is measured at $x = 100$ m. The parameters are $\mu = 3$, $h_0 = 50$ m, speed limit = 32 m/s, $d_{\mathrm{merge}} = 300$ m, driver reaction time $t_d = 0.75$ s, and the mechanical time