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Optimization of network structure to random failures

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Abstract

Network's resilience to the malfunction of its components has been of great concern. The goal of this work is to determine the network design guidelines, which maximizes the network efficiency while keeping the cost of the network (that is the average connectivity) constant. With a global optimization method, memory tabu search (MTS), we get the optimal network structure with the approximately best efficiency. We analyze the statistical characters of the network and find that a network with a small quantity of hub nodes, high degree of clustering may be much more resilient to perturbations than a random network and the optimal network is one kind of highly heterogeneous networks. The results strongly suggest that networks with higher efficiency are more robust to random failures. In addition, we propose a simple model to describe the statistical properties of the optimal network and investigate the synchronizability of this model.

Keywords: Complex network; Network efficiency; Network resilience; Synchronization

1. Introduction

Complex networks arisen in natural and man-made systems play an essential role in modern society. Many real complex networks were found to be heterogeneous with power-law degree distributions: $p(k) \sim k^{-\gamma}$, such as the Internet, metabolic networks, scientific citation networks, and so on [1,2]. Because of the ubiquity of scale-free networks in natural and man-made systems, the security of these networks, i.e., how well these networks work under failures or attacks, has been of great concern.

Recently, a great deal of attention has been devoted to the analysis of error and attack resilience of both artificially generated topologies and real-world networks [3–11]. Also some researchers use the optimization approaches to improve the network's robustness with percolation theory or information theory [12–17]. There are various ways in which nodes and links can be removed, and different networks exhibit diverse levels of resilience to such disturbances. It has been pointed out by a number of authors [3–7] that scale-free networks

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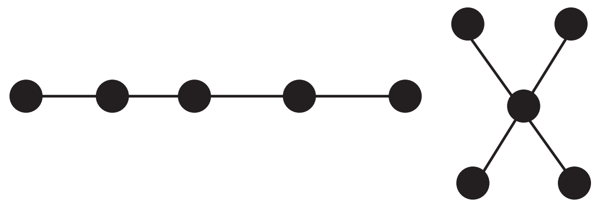

Fig. 1. (a) $\langle k\rangle = 8/5, E = 8/25$; (b) $\langle k\rangle = 8/5, E = 7/20$.

are resilient to random failures, while fragile to intentional attacks. That is, intentional attack on the largest degree (or betweeness) node will increase the average shortest path length greatly. While random networks show similar performance to random failures and intentional attacks.

The network robustness is usually measured by the average node–node distance, the size of the largest connected subgraph, or the average inverse geodesic length named *efficiency* as a function of the percentage of nodes removed. Efficiency has been introduced in the studies of small-world networks [18] and used to evaluate how well a system works before and after the removal of a set of nodes [6].

The network structure and function strongly rely on the existence of paths between pairs of nodes. Different connectivity pattern between pairs of nodes makes the network different performance to attacks. Rewiring edges between different nodes to change the topological structure may improve the network's function. As an example, consider the simple five nodes network shown in Fig. 1. The efficiency of Fig. 1(a) is equal to $\frac{8}{25}$, while it is improved to $\frac{7}{20}$ in Fig. 1(b) by rewiring. And we know that Fig. 1(b) is more robust than Fig. 1(a) to random failures. A natural question is addressed: how to optimize the robustness of a network when the cost of the network is given. That is, the number of links remains constant while the nodes connect in a different way. Should the network have any particular statistical characters? This question motivates us to use a heuristic approach to optimize the network's function by changing the network structure.

The paper is organized as follows: we firstly present memory tabu search (MTS) method in Section 2 and the numerical results are shown in Section 3. Then we construct a simple model to describe the optimal network and discuss one of the important dynamic processes happening on the network, synchronization, in Section 4. Finally, we give some insightful indications in Section 5.

2. The algorithm

Generally, a network can be described as an unweighted, undirected graph G. Such a graph can be presented by an adjacency binary matrix $A = \{a_{ij}\}$. $a_{ij} = 1$ if and only if there is an edge between node i and j. Another concerned matrix $D = \{d_{ij}\}$, named distance matrix, consists of the elements denoting the shortest path length between any two different nodes. Then the efficiency ε_{ij} between nodes i and j can be defined to be inversely proportional to the shortest distance: $\varepsilon_{ij} = 1/d_{ij}$ [18]. The global efficiency of the network is defined as the average of the efficiency over all couples of nodes

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \varepsilon_{ij} = \frac{1}{N(N-1)} \sum_{i \neq j \in G} \frac{1}{d_{ij}}. \tag{1}$$

With the above robustness criterion in mind, we can define the optimization problem as follows:

$$\begin{cases} \max & E(G) \\ \text{s.t.} & \langle k\rangle = \text{const.} \\ & G \text{ is connected.} \end{cases} \tag{2}$$