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Epidemic spreading in communities with mobile agents

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ABSTRACT

We propose a model of mobile agents to study the epidemic spreading in communities with different densities of agents, which aims to simulate the realistic situation of multiple cities. The model addresses the epidemic process from a community with threshold λ_{c1} less than the infection rate λ to a community with threshold λ_{c2} larger than λ through both direct and indirect contacts. By both theoretic analysis and numerical simulations we show that it is possible to sustain the epidemic spreading in the community with λ_{c2} through contact with another community, provided that the latter is connected with an infected community. This result suggests that for effectively controlling the epidemic spread, we should also pay attention to the risk caused by the infection through indirect contact.

1. Introduction

Epidemic spreading has been studied for a long time and recently attracted great interest because of the new discoveries in complex networks [1–3]. One of the key problems in this field is to find effective approaches to protect human communities from infection. A number of methods have been proposed to solve this problem, especially the segregate method [4–8], which is to cut the connection between the infected people (communities) and non-infected ones. The segregate method is a good choice when we do not know the property of the diseases or we do not have an effective way to immunize ourselves. It is generally believed that a community is safe if its connections to the surrounding infected communities are removed [4,6]. However, real situations are very complex. On one hand, there is a time delay between the infection of an agent and the knowledge of its status by others, which causes some difficulties to timely figure out the dangerous connections. On the other hand, the epidemic spread may go through a third one, i.e., a safe community with an epidemic threshold λ_c smaller than its current infection rate. As we know, a single individual, can temporarily immunize himself or cut all his connections with the outside to protect himself once he is in dangerous environment. However, for a community, it is hard to do that as it has a large amount of transportation/communication with the outside to sustain its normal function. Considering the fact that connections with the outside will always bring some risk, in this paper, we will build a dynamical community model and make a theoretical analysis to understand how large the risk is. This problem is significant as it will help the governor to make an optimal strategy to balance the risk and the necessary transportation/communication with the outside.

Firstly, we need to understand how epidemics spread between two connected communities. This problem has been recently addressed in static community networks by Newman et al. [9,10] and Liu et al. [11] where the community structures are fixed and the viruses/diseases can only spread from one community to another through the links between them. Considering the fact that human beings in a community are mobile agents [12–16] and each agent may travel to other communities by transport, the links in and between communities are dynamical but not static. Therefore, it is necessary to investigate the epidemic spreading in dynamical communities. In this paper, a dynamical community model for the two

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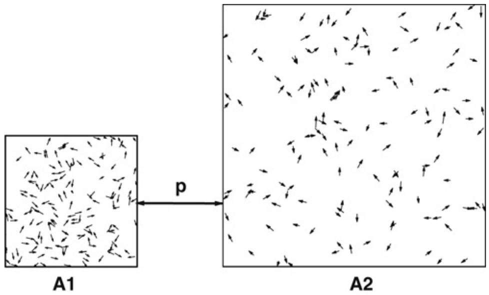

Fig. 1. Schematic illustration of the epidemic spreading in the dynamical community model where the small segments with arrow denote the moving individuals and the bridge between the two communities represents the jumping of agents.

and three connected communities is presented. The model shows that: (1) There is a stabilized number of infected agents after a transient process in each community, which depends on its density of agents and the jumping probability between two neighboring communities; (2) In the case of two connected communities, a parameter γ is introduced to represent the efficiency of the jumping agents to reproduce infectors. It is found that the ratio of the number of infected agents between the two communities is inversely proportional to the average degree of the safe community; (3) In the case of three communities, the ratio of the number of infected agents is inversely proportional to the product of degrees of the two safe communities.

An epidemic process can be generally characterized by the SIS (susceptible-infected-susceptible) model [17–20] and the SIR (susceptible-infected-refractory) model [21–26]. In the SIS model, an agent has two status: susceptible and infected. A susceptible agent may become infected once it contacts an infected one. After a time step, the infected agent recovers and returns to the susceptible state. In this model, an interesting result pointed out by Paster–Satorras and Vespignani is that the virus could spread on the Internet and WWW even when the infection probabilities are vanishingly small [17]. While in the SIR model, an agent has three status: susceptible, infected, and refractory. The infected agent cannot go back to a susceptible status but become refractory, which describes the phenomenon of long-term immunity. In this paper, we use the SIS model.

The paper is organized as follows. In Section 2, we propose the dynamical community model. In Section 3, we discuss the epidemic spreading between two connected networks. Both theoretic analysis and numerical simulations are given. In Section 4, we discuss the epidemic spreading through indirect contagion in three communities. We find that for properly chosen population densities, it is possible to sustain the epidemic in a third community through indirect contact. Finally, the conclusions are given in Section 5.

2. The dynamical community model

Considering the fact that an agent can freely move in or between communities, whose neighbors change with time [12–16], we may conceive of a dynamical community model that contains the above features. Suppose an agent interacts only with its neighbors within radius r, i.e., there are links between those agents whose distance is smaller than $2r$. Thus, the links are time dependent and the degree k of an agent is determined by the number of agents within the circle of radius r. Also noticing that agents may sometimes travel to other communities, we let each agent have a probability p to jump to a neighboring community. In static community networks [9–11], this jumping behavior is equivalent to the links between communities. Obviously, the difference between the static and dynamical models is that these links are only for the chosen specific agents in the static community network but be possible for every agent in the dynamical community model. Therefore, the dynamical model is closer to reality. Considering the fact that the jumping process is much shorter than the infection period, we assume that the status of the jumping agents will be kept when it jumps to another community. Based on this analysis, our model consists of multiple communities with jumping agents between any two of them. Fig. 1 shows the schematic illustration of two communities where each one is a square-shaped area with a periodic boundary condition, i.e., in a two-dimensional space. In contrast to the real world, these areas could represent different cities that have a long geographical distance between them. People can move freely in their own city and travel to other cities by transport, such as planes etc.

Suppose there are N agents who move with velocity $\mathbf{v}(t)$ in each community. The modulus of the velocity of all the agents v are the same in the whole spreading process while their directions are randomly distributed and updated stochastically in time. That is, each agent is a random walker with a probability p to travel to another community and the probability $1-p$ to stay in the community. As both A_1 and A_2 have the same population and the same jumping probability, their population will be always kept as N. When an agent travels to another community, its new position will be randomly chosen. On the contrary, its position will be updated with the following relation if it moves within the same community

$$\mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t, \tag{1}$$

where $\mathbf{x}_i(t)$ is the position of the ith agent in the two-dimensional space at time t, $\mathbf{v}_i(t) = (v\cos\theta_i(t), v\sin\theta_i(t))$, $\theta_i(t) = \xi_i(t)$, $\xi_i(t)$ are N independent random variables chosen at each time with uniform probability in the interval $[-\pi, \pi]$, and the velocity amplitude v is chosen as 0.03 in this paper. In the limit $v \to 0$ the agents do not move and