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A model for cascading failures in scale-free networks with a breakdown probability

Jian-Wei Wang *, Li-Li Rong

Institute of Systems Engineering, Dalian University of Technology, 2 Ling Gong Road, Dalian 116024, Liaoning, PR China

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ABSTRACT

Considering that not all overload nodes will be removed from networks due to some effective measures to protect them, we propose a new cascading model with a breakdown probability. Adopting the initial load of a node j to be $L_j = [k_j(\sum_{m\in\Gamma_j} k_m)]^\alpha$ with k_j and Γ_j being the degree of the node j and the set of its neighboring nodes, respectively, where α is a tunable parameter, we investigate the relationship between some parameters and universal robustness characteristics against cascading failures on scale-free networks. According to a new measure originated from a phase transition from the normal state to collapse, the numerical simulations show that Barabási–Albert (BA) networks reach the strongest robustness level against cascading failures when the tunable parameter $\alpha = 0.5$, while not relating to the breakdown probability. We furthermore explore the effect of the average degree $\langle k\rangle$ for network robustness, thus obtaining a positive correlation between $\langle k\rangle$ and network robustness. We then analyze the effect of the breakdown probability on the network robustness and confirm by theoretical predictions this universal robustness characteristic observed in simulations. Our work may have practical implications for controlling various cascading-failure-induced disasters in the real world.

1. Introduction

Cascading failures are common phenomenon in real-life systems and can occur in many infrastructure networks, including the electrical power grid, the Internet, transportation networks, and so on. Evidence has demonstrated that in such networks, even though most failures emerge very locally, the entire network can be largely affected, even resulting in global collapse. Typical examples are several largest blackout events [1] in some countries and Internet congestions [2]. Therefore, cascading failures have been of great concern and widely investigated recently, with a major focus on scale-free networks for their ubiquity in natural and human-made systems.

A number of important aspects of cascading failures in complex networks have been discussed in the literature and many valuable results have been found, including the cascade phenomenon triggered by intentional attacks [3–8], avalanche size distributions [9–11], congestion instabilities [12–16], the cascade control and defense strategy [17–22], the model for describing cascade phenomena [23–31], and so on. Recently, many scholars have investigated cascading failures based on the different definitions of the initial load of a node. Wang et al. [11] discussed the universal robustness characteristic of weighted networks against cascading failures by adopting the initial load of an edge to $(k_ik_j)^\theta$, with k_i and k_j being the degrees of the nodes connected by the edge. Following the work of Wang et al., Wu et al. [29] used a similar method to assign the initial load of a node and studied the onset and spreading of cascading failures on weighted heterogeneous networks. In addition, according to the relationship between the load and the betweenness of a node, Sun et al. [21] proposed

* Corresponding author. Tel.: +86 411 81258693.
E-mail address: wdut@yahoo.cn (Jian-Wei Wang).

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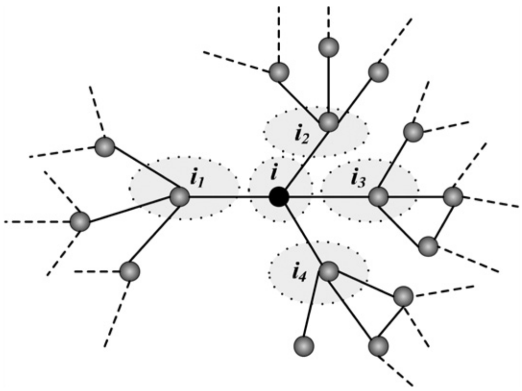

Fig. 1. A scheme illustrating the correlation between the load of a node and its degree and the total degrees of its neighboring nodes. Generally, the bigger the degree of node i and the total degrees of its neighboring nodes i_1, i_2, i_3, and i_4, the greater the load of node i.

a new matching model of capacity by developing a profit function to defend againstcascading failures on artificially created scale-free networks and the real network structure of the North American power grid. In all studies cited above, the load on a node (or an edge) was generally estimated by its degree or its betweenness. However, the degree method is inferior owing to its consideration of only a single node degree, which loses much information in many actual applications; the betweenness principle, however, is only practical for small and medium-sized networks but invalid for large scale ones such as the Internet or World Wide Web, due to its consideration of topological information for the whole network. Moreover, in most previous cascade failure models, the simple strategy of immediate removal of an instantaneously overloaded node is widely adopted, without considering the case that not all overload nodes will be removed from the network owing to certain monitoring and effective measures in most real-life infrastructure networks.

Therefore, taking the above two methods into account, the main differences of our cascading model from previous studies are as follows:

(1) To reduce the complexity of the betweenness and improve the practicability of the degree in a cascading study, we define the initial load of a node depending on its degree and the total degrees of its neighboring nodes. Fig. 1 illustrates the correlation between the load of a node and its degree and the total degrees of its neighboring nodes.

(2) We propose a new concept of the breakdown probability of an overload node, which represents the probability that an overload node will be removed from networks.

Taking the robustness to be quantified by the critical threshold β_c, at which a phase transition occurs from normal state to collapse, we discuss the universal cascading phenomenon on a typical network, i.e., a Barabási–Albert (BA) network [32] with scale-free properties. The simulation results show the relationship between the robustness level of scale-free networks against cascading failures and some parameters in our cascading model. In addition, we also verify numerical results from the theoretical analysis. Our findings may be useful in furthering studies of control of and defense against cascading failures in many real-life complex networks.

The rest of this paper is organized as follows: in Section 2, we describe the cascading model in detail. The parameters in our model are discussed based on BA networks in Section 3. In Section 4, the numerical results are verified by the theoretical analysis. Finally, some summaries and conclusions are given in Section 5.

2. Cascading failure load model

Our studies only focus on the dynamic properties of the network, showing that the removal of one node can have important consequences. Initially the network is in a stationary state in which the load at each node is smaller than its capacity. The removal of a node will change the balance of the load and lead to redistribution of the load over other nodes. If the capacity of these nodes is insufficient to handle the extra load, this will be redistributed in turn, triggering a cascade of overload failures and eventually a large drop in the performance of the network, such as the Internet or electrical power grids.

We therefore propose a cascading load model. We will first introduce the initial load assignment of a node and the redistribution rule of the load in detail.

According to Fig. 1, we assume the initial load of a node j to be $L_j = [k_j(\sum_{m\in\Gamma_j} k_m)]^\alpha$ with k_j and Γ_j being the degree of the node j and the set of its neighboring nodes, respectively, where α is a tunable parameter and governs the strength of the node load. The load of a broken node i will be redistributed to its neighboring node j, depending on the preferential probability: $\Pi_j = [k_j \sum_{m\in\Gamma_j} k_m]^\alpha / \sum_{n\in\Gamma_i}[k_n \sum_{f\in\Gamma_n} k_f]^\alpha$. Because the node capacity in real-life networks is generally limited by cost, it is natural to assume that the capacity C_j of the node j is proportional to its initial load for simplicity: $C_j = \beta * L_j, j = 1, 2, 3, \ldots, N$, where the constant $\beta(\geq 1)$ is a tolerance parameter.