Physica A 405 (2014) 10–16

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Time delay induces oscillatory coherence in intracellular calcium oscillation system☆

Wei-Long Duan*

City College, Kunming University of Science and Technology, Kunming 650051, China

HIGHLIGHTS

- Both NAFs exhibit periodically synchronous sawtooth waves.
- Sawtooth waves weaken as correlation time of colored noises or autocorrelation time increases.
- Time delay induces oscillatory coherence in intracellular calcium oscillation.

ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form 17 February 2014
Available online 10 March 2014

Keywords:
Intracellular calcium oscillation
Time delay
Colored noise
Coherence resonance

ABSTRACT

In view of same time delay τ in active and passive transmission processes with colored noises of intracellular Ca^{2+}, by means of second-order algorithm for stochastic simulation colored noises, the normalized autocorrelation function (NAF) and the characteristic correlation time (CCT) of cytosolic and calcium store's Ca^{2+} concentration are studied. For the case of long τ: both NAFs of cytosolic and calcium store's Ca^{2+} concentration exhibit periodically synchronous sawtooth wave, which linearly weakens as correlation time of colored noises or autocorrelation time increases. As τ increases, time delay induces oscillatory coherence in intracellular calcium oscillation system.

1. Introduction

In many studies on intracellular calcium oscillation (ICO), there are a variety of channels showing calcium-induced calcium release and a variety of models to describe ICO as in Refs. [1–4]. Many interesting phenomena have been found such as stochastic resonance [5], coherence resonance [6,7], bistability solutions with hysteresis [8,9], calcium puffs [10], various spontaneous Ca^{2+} patterns [11], non-Gaussian noise-optimized ICO in cytosol [12], stochastic backfiring [13], and dispersion gap and localized spiral waves [14]. This is an interesting topic which has been intensively studied by Falcke group [13–18], they have given that Ca^{2+} is a ubiquitous second messenger. Importantly, experimentally measured calcium oscillation is inherently stochastic. Careful analysis of experimental data and comparison with mathematical models showed clearly that real ICO is non-Gaussian [18].

More importantly, Matjaž Perc group [19–24] has studied the effects of noise on the dynamics of calcium oscillation. It shown directly that noise and other stochastic effects indeed play a central role [19,20] from experimental data. In fact, stochasticity is a predominant mode of functioning in intracellular processes [25]. For instance, noise may be beneficial for the stability and robustness of calcium oscillation [21]. At the same time, noise could aid the detection of weak calcium

☆ This project was supported by the National Nature Science Foundation of China (Grant Nos. 11347014, 11305079 and 11347101).
* Tel.: +86 15087195310.
E-mail address: yndwl@sina.com.cn.

http://dx.doi.org/10.1016/j.physa.2014.03.019
0378-4371/

signals within the cell [22] and could induce periodic calcium waves [24]. Moreover, they have demonstrated that the effects of noise become lesser [23] when the cells are coupled. In a word, Matjaž Perc group mainly studied the effect of white noise on ICO system, rather than colored noise.

However, colored noise is widespread in many stochastic dynamical systems including biological cells, rather than ideally white noise. In our recent research on ICO [9,26–28], when considering white noise the time evolutions of Ca^{2+} concentrations exhibit strange changes which is not reasonable in a real system, thus only taking into account colored noise might reasonably explain the ICO phenomenon. Moreover, in research on ICO, almost all researchers neglected time delay in transmission processes of intracellular Ca^{2+}. However, inclusion of time delay is natural due to finite transmission speed of matter, energy and information. Therefore, in the ICO system, we consider time delay in processes of active and passive transport of intracellular Ca^{2+}, because both processes must take finite time to complete. It is worth noting that in the early references such as [29–33] authors have studied the role of time delay on stochastic dynamics systems and found many interesting phenomena. In simulating time series, it shows that the role of time delay of active transport is same as the role of time delay of passive transport, and both time delays tend to equal. In order to study easily, in this paper we suppose that both processes take same time.

Anyway, in the ICO system, same time delay in both processes and colored noises are considered. Interestingly, the time series of cytosolic and calcium store's Ca^{2+} oscillation show periodically anti-synchronous oscillation, and time series exhibit regular periodic oscillation when taking certain parameters of time delay and colored noises, but time series will also exhibit irregular oscillation when taking certain parameters of time delay and colored noises. Namely, the regularity of time series will change as parameters of time delay and colored noises vary. Thus, studying the effect of time delay and colored noises on autocorrelation property of cytosolic and calcium store's Ca^{2+} oscillation is significant. In this paper, we define functions to study these property.

In Section 2, the model for ICO in view of colored noises and time delay in processes of active and passive transport is presented based on Refs. [6,9,27,28]. Then a second-order algorithm for stochastic simulation colored noises of the ICO system is introduced in Section 3. NAF and CCT of cytosolic and calcium store's Ca^{2+} concentration are respectively introduced and simulated in Section 4. Finally, conclusions are drawn in Section 5.

2. The model for ICO with time delay and colored noises

In view of same time delay τ in processes of active and passive transport of Ca^{2+} in a real cell, the Langevin equations of ICO system read as follows according to Ref. [27]:

$$d_t x = A_1(x, y; x_\tau, y_\tau) + B_1(x, y; x_\tau, y_\tau)\epsilon(t), \tag{1}$$

$$d_t y = A_2(x, y; x_\tau) + B_2(x, y; x_\tau)\Gamma(t), \tag{2}$$

with

$$A_1(x, y; x_\tau, y_\tau) = v_0 + v_1\beta_0 - v_2 + v_{3\tau} + k_f y_\tau - kx, \tag{3}$$

$$A_2(x, y; x_\tau) = v_{2\tau} - v_3 - k_f y, \tag{4}$$

$$B_1(x, y; x_\tau, y_\tau) = \sqrt{v_1^2\beta_0^2 + 2v_1\beta_0\lambda W + W^2}, \tag{5}$$

$$B_2(x, y; x_\tau) = \sqrt{\frac{1}{V}(v_{2\tau} + v_3 + k_f y)}, \tag{6}$$

$$W(x, y; x_\tau, y_\tau) = \sqrt{\frac{1}{V}(v_0 + v_1\beta_0 + v_2 + v_{3\tau} + k_f y_\tau + kx)}, \tag{7}$$

and

$$v_2 = \frac{V_2 x^2}{x^2 + k_1^2}, \qquad v_3 = \frac{V_3 x^4 y^2}{(x^4 + k_2^4)(y^2 + k_3^2)}, \tag{8}$$

$$v_{2\tau} = \frac{V_2 x_\tau^2}{x_\tau^2 + k_1^2}, \qquad v_{3\tau} = \frac{V_3 x_\tau^4 y_\tau^2}{(x_\tau^4 + k_2^4)(y_\tau^2 + k_3^2)}. \tag{9}$$

Here, x and y denote the concentration of free Ca^{2+} of cytosol and calcium store in a cell, respectively. The rates v_2 and v_3 refer, respectively, to pumping of Ca^{2+} into the calcium store and to release of Ca^{2+} from the store into the cytosol in a process activated by cytosolic Ca^{2+}. $v_{2\tau}$ is v_2 with time delay, and $v_{3\tau}$ is v_3 with time delay. $W = W(x, y; x_\tau, y_\tau)$, $x_\tau = x(t-\tau)$, $y_\tau = y(t-\tau)$. λ denotes the cross-correlation degree of internal and external noise before merger [9]. The value of parameters is set as in Ref. [6]: $v_0 = 1\ \mu M/s$, $v_1 = 7.3\ \mu M/s$, $\beta_0 = 0.287$, $k_f = 1/s$, $k = 10/s$, $V_2 = 65\ \mu M/s$, $V_3 = 500\ \mu M/s$, $k_1 = 1\ \mu M$, $k_2 = 0.9\ \mu M$, $k_3 = 2\ \mu M$ and $V = 1000$.