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A Bohr-type model of a composite particle using gravity as the attractive force

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HIGHLIGHTS

- We model the bound states formed by gravitationally confined relativistic particles.
- We use special relativity with Newton's gravitational law or general relativity.
- We use the Bohr methodology for quantization of the angular momentum.
- The mass of states formed by three relativistic neutrinos is in the hadrons range.
- Binding and free energies of the composite states are in good agreement with QCD.

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ABSTRACT

We formulate a Bohr-type rotating particle model for three light particles of rest mass m_o each, forming a bound rotational state under the influence of their gravitational attraction, in the same way that electrostatic attraction leads to the formation of a bound proton–electron state in the classical Bohr model of the H atom. By using special relativity, the equivalence principle and the de Broglie wavelength equation, we find that when each of the three rotating particles has the same rest mass as the rest mass of a neutrino or an antineutrino ($\sim$0.05 eV/c^2) then surprisingly the composite rotating state has the rest mass of the stable baryons, i.e. of the proton and the neutron ($\sim$1 GeV/c^2). This rest mass is due almost exclusively to the kinetic energy of the rotating particles. The results are found to be consistent with the theory of general relativity. The model contains no unknown parameters, describes both asymptotic freedom and confinement and also provides good agreement with QCD regarding the QCD condensation temperature. Predictions for the thermodynamic and other physical properties of these bound rotational states are compared with experimental values.

1. Introduction

The semiclassical Bohr model for the H atom, first presented a century ago [1], provides quantitative description of all the basic properties of the H atom. In this model one utilizes both the corpuscular and the ondular (wave) nature of the rotating electron. Indeed, by considering the corpuscular nature of an electron of mass m_e, Newton's second law for a circular particle motion implies

$$F = m_e \mathrm{v}^2/R. \tag{1}$$

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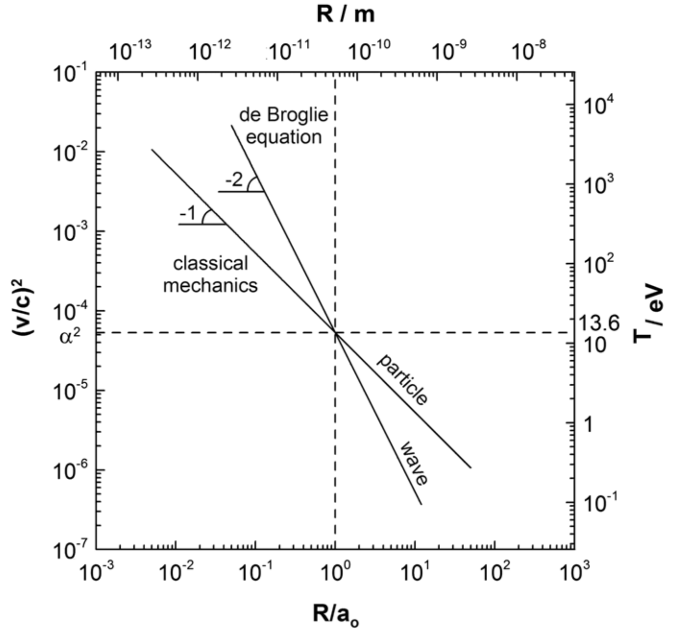

Fig. 1. Graphical solution of the two equations of the Bohr model, i.e. of the classical mechanical equation $v^2 = e^2/\epsilon m_e R$ (Eq. (3)) and of the de Broglie wavelength equation, for $n = 1$, $v^2 = \hbar^2/m_e^2 R^2$ (Eq. (4)). The kinetic energy T is computed from $T = (1/2)m_e v^2$, a_o is the Bohr radius $\hbar/m_e\alpha c$.

Assuming that the force F is described by Coulomb's law, i.e.,

$$F = e^2/\epsilon R^2. \tag{2}$$

Eq. (1) yields

$$R = e^2/\epsilon m_e v^2. \tag{3}$$

The additional equation needed to obtain v and R is obtained by utilizing the ondular nature of the electron, viewed as a standing wave, via the de Broglie wavelength expression

$$\lambdabar = R = \frac{n\hbar}{m_e v}, \tag{4}$$

where $\lambdabar$ is the reduced de Broglie wavelength (assumed to be equal to the rotational radius R), n is a positive integer and $\hbar$ is the reduced Planck constant.

Upon combining (1)–(4) one obtains the following well known formulas:

$$v/c = \frac{e^2}{n\epsilon c\hbar} = \frac{\alpha}{n}, \tag{5}$$

$$R = \frac{n^2\hbar}{\alpha m_e c} = n^2 a_o, \tag{6}$$

$$\mathcal{H} = -\frac{1}{2n^2}\left[\alpha^2 m_e c^2\right], \tag{7}$$

where $\alpha(\approx 1/137.035)$ is the fine structure constant, a_o is the Bohr radius and $\mathcal{H}$ is the Hamiltonian.

A graphical solution of Eqs. (3) and (4) is given in Fig. 1, which underlines that in the Bohr model both the corpuscular and the ondular nature of the electron are considered, the latter expressed via the de Broglie wavelength equation. This equation played a crucial role in the development of quantum mechanics.

Although the deterministic Bohr model description of the H atom has been gradually replaced by the quantum mechanical Schrödinger equation, and is used today mostly for pedagogical purposes, it is worth remembering that the Bohr model (as well as its Bohr–Sommerfeld elliptical orbit extension [2]), leads to the same level of quantitative description as the Schrödinger equation for all the basic properties of the H atom.

A natural variation of Bohr's model described by Eqs. (3) and (4), is to replace the electrostatic attraction by gravity and to examine to what, if any, system such a model may be related to. Thus, as an example, one may consider three light particles