Physica A 405 (2014) 35–51

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A study on modeling the dynamics of statistically dependent returns

Hamed Davari-Ardakani, Majid Aminnayeri *, Abbas Seifi

Department of Industrial Engineering and Management Systems, Amirkabir University of Technology, P.O. Box 15875-4413, Tehran, Iran

HIGHLIGHTS

- The dynamic behavior of financial data is modeled through a set of scenarios.
- Statistical properties and marginal distributions of historical data are preserved.
- A scenario based stochastic optimization model is implemented with the scenario set.
- High value of stochastic model and in-sample stability of the objective is confirmed.
- Out-of-sample simulations show the outstanding performance of the proposed method.

ARTICLE INFO

Article history:
Received 27 October 2013
Received in revised form 23 February 2014
Available online 6 March 2014

Keywords:
Scenario set
Heteroskedastic time series
Serial correlation
Statistical dependence
Multi-period portfolio

ABSTRACT

This paper develops a method to characterize the dynamic behavior of statistically dependent returns of assets via a scenario set. The proposed method uses heteroskedastic time series to model serial correlations of returns, as well as Cholesky decomposition to generate the set of scenarios such that the statistical dependence of different asset returns is preserved. In addition, this scenario generation method preserves marginal distributions of returns. To demonstrate the performance of the proposed method, a multi-period portfolio optimization model is presented. Then, the method is implemented through a number of stocks selected from New York Stock Exchange (NYSE). Computational results show a high performance of the proposed method from the statistical point of view. Also, results confirm sufficiency and in-sample stability of the generated scenario set. Besides, out-of-sample simulations, for both risk and return, illustrate a good performance of the proposed method.

1. Introduction

The evolution of financial markets enforces individual and organizational investors to face with high degrees of uncertainty. Consequently, taking timely and well informed decisions is a matter of particular importance. Hence, the involved uncertainties should be modeled properly. In this paper, a scenario generation approach is proposed to deal with this important issue. The generated scenario set should preserve statistical features of the reference data set. Moreover, it should perform well in practical cases.

Høyland and Wallace [1] presented a nonlinear programming model for scenario generation. They generated a number of scenarios, such that some distance measures between the specified statistical properties and those of generated scenarios are

* Corresponding author. Tel.: +98 21 66413034; fax: +98 21 66954569.
E-mail addresses: hameddavari@aut.ac.ir (H. Davari-Ardakani), mjnayeri@aut.ac.ir (M. Aminnayeri), aseifi@aut.ac.ir (A. Seifi).

http://dx.doi.org/10.1016/j.physa.2014.02.077
0378-4371/

minimized. This method is referred to as "moment matching". A remarkable number of studies used the moment matching method individually or in combination with other methods in their work, Refs. [2–7].

Experiences show that distributions of asset return are not necessarily uni-modal. Therefore, matching a small number of moments, especially in case of multi-modal return distributions, does not seem to well preserve marginal distributions of asset returns. In addition, to the best of our knowledge, all studies that use moment matching, utilize covariance matrices to measure the dependence of return series. Since covariance matrices only measure the linear dependency of random variables for normal return distributions, they do not seem to be a confident tool, when return distributions are non-normal. Therefore, they may give misleading results.

A few research works focused on serial dependence of return series, which in turn is an important issue in asset returns. Ji et al. [5] and Grebeck & Rachev [8] used Vector Auto-Regression (VAR) method to model random variables as time series for financial scenario generation. Also, they used an exponentially weighted moving average (EWMA) process to remove the major trends of the individual time series. Erlwein et al. [9] utilized the hidden Markov model to deal with scenario generation of financial parameters. Although these studies consider serial correlations of return series, they do not consider heteroskedastic and leptokurtic behavior of asset returns. The former means that their variance is not constant and the latter means that distributions of returns have fatter tails than normal distributions. These characteristics distinguish financial time series data from other types of data series. Therefore, utilizing appropriate time series models is necessary to account for the intrinsic characteristics of financial data series in scenario generations. The autoregressive conditional heteroskedastic (ARCH) model of volatility was presented by Engle [10] and further developed to the generalized ARCH (GARCH) model by Bollerslev [11]. For an elaborate discussion about conditional heteroskedastic models, one can refer to Tsay [12]. There are a limited number of works that considered the above-mentioned behavior of financial data, and utilized heteroskedastic models to generate financial scenarios. Chen and Yuen [13], Chen et al. [14] and Chen [15] presented a scenario generation approach to deal with the uncertain behavior of asset returns. Their method consists of two steps. First, they used a GARCH-type process to model returns of risky assets. Second, they used the conditional sampling to get the return value at each node of the scenario tree and the conditional probability for a scenario to pass through a branch at a node.

In this paper, we use different GARCH type models in combination with ARMA models to model the serial dependence of financial data series. Moreover, it should be considered that marginal distributions of financial data series are often non-normal. As mentioned before, most of studies assumed that marginal distributions of data series follow a normal distribution. We suggest using Johnson transformation [16] for converting non-normal distributions of asset return to normal ones. By using Johnson transformation, we can transform distributions of return series, which in some cases are unknown, to the standard normal distribution. This has three advantages. First, we can use special characteristics of normal distribution, such as its special linear dependence structure. Second, it dispels our concerns about fat tails of return distributions. Finally, it helps decision maker(s) fit time series models on return series with Gaussian innovations. Of course, after forming the set of scenarios, all of simulated returns should be back transformed to original distributions of returns.

In addition, we try to exclude arbitrage opportunities from the set of generated scenarios as much as possible to make it consistent with conditions of real markets and asset pricing theory.

The discussion about diversity of scenario generation methods is not limited to above-mentioned studies, and we have mentioned some key points of this area. See for example [17,18]. For a more detailed discussion about scenario generation techniques, one can refer to Ref. [19].

In summary, this paper develops a scenario generation method that not only takes the dependence structure of financial data into account, but also it considers serial correlations of data series. Since financial data series, e.g. asset returns, often exhibit heteroskedasticity, ARMA/GARCH type models are used to model the time dependence of financial data series. To overcome the non-normality and fat-tailed distribution of return series, Johnson transformation is utilized. This transformation has also the main advantage that it enables us to generate scenarios without identification of marginal distributions of data series. This makes the proposed method more accurate, since it eliminates the errors associated with fitting marginal distributions on the univariate data series. Also, Cholesky decomposition is used to model the dependence structure of return series. Finally, the preclusion of arbitrage opportunities is investigated.

The remainder of this paper is organized as follows. Section 2 presents the theoretical background of the proposed scenario generation method. The proposed scenario generation method and a multi-period portfolio selection model are presented in Section 3. Numerical and graphical results are presented and discussed in Section 4. Finally, Section 5 concludes the paper.

2. Theoretical background

In this section, we discuss some important theoretical aspects of the proposed scenario generation method.

2.1. Transformation of non-normal returns to normal distribution

To exploit the properties of normal distribution in modeling the dependence structure of returns, Johnson transformation [16] is utilized. This helps us exploit the covariance matrix for modeling the dependence structure of returns, and dispels our concerns about fat tails of distributions. Johnson transformation is of three types referred to as bounded system (S_B), log-normal system (S_L) and unbounded system (S_U). Eqs. (1)–(3) show these three types of transformations respectively, where