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# Nonlinear dynamics of autonomous vehicles with limits on acceleration

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## HIGHLIGHTS

- Stability of platoon with limits on acceleration determined.
- Small-deviation string stability insufficient to avoid collisions.
- No vehicle exceeds limits if lead vehicle remains within limits.
- Initial equilibrium state with proper control law parameters required.
- Optimal acceleration feedback gain varies with engine speed.

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## ABSTRACT

The stability of autonomous vehicle platoons with limits on acceleration and deceleration is determined. If the leading-vehicle acceleration remains within the limits, all vehicles in the platoon remain within the limits when the relative-velocity feedback coefficient is equal to the headway time constant [$k = 1/h$]. Furthermore, if the sensitivity $\alpha > 1/h$, no collisions occur. String stability for small perturbations is assumed and the initial condition is taken as the equilibrium state. Other values of $k$ and $\alpha$ that give stability with no collisions are found from simulations. For vehicles with non-negligible mechanical response, simulations indicate that the acceleration-feedback-control gain might have to be dynamically adjusted to obtain optimal performance as the response time changes with engine speed. Stability is demonstrated for some perturbations that cause initial acceleration or deceleration greater than the limits, yet do not cause collisions.



## 1. Introduction

Autonomous vehicles or adaptive cruise control (ACC) vehicles could be a significant factor in future transportation systems. Various authors have shown that if ∼30% of vehicles had ACC, the formation of jams in heavy traffic could be eliminated [1–10]. The ACC vehicles are assumed to be governed by control parameters that give string stability [11].

As the literature on string stability is large, no attempt to summarize all the papers will be given here. The reader is referred to several good references on traffic and ACC [12–16]. However, I will describe the salient developments as they relate to the present work. The first is the constant headway time policy that requires the control system to maintain the headway between two vehicles as $hv$, where $h$ is the headway time constant (also known as the time gap, which refers to bumper-to-bumper gap) and $v$ is the velocity [17–19].

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The control algorithm for a vehicle's acceleration is (assuming instantaneous mechanical response) $a = \frac{\alpha}{h}(\Delta x - D - hv) + k\Delta v$, where $\alpha$ is the sensitivity, $k$ is the gain for relative velocity feedback control, $D$ is the length of the vehicle plus a safety margin, $\Delta x$ is the center-to-center distance between vehicles, and $\Delta v$ is relative velocity $\frac{\mathrm{d}\Delta x}{\mathrm{d}t}$ [20]. Liang and Peng showed that string stability is attained if $\alpha + 2k > \frac{2}{h}$ [11].

If the mechanical response of the vehicle is characterized by a first-order time constant $\tau$, the maximum allowed $\tau$ occurs when $k = \frac{1}{h}$ independent of $\alpha$ and is given by $\tau_{\max} = \frac{h}{2}$ [20]. String stability has also been analyzed for an explicit delay time $t_d$ by Orosz, Moehlis and Bullo [21] as well as for more general mechanical responses [22,23]. To date, however, the effects of comfortable limits on acceleration and deceleration have not been examined in depth. The purpose of this paper is to examine platoon stability with such limits.

The only comparable model that has maximum acceleration and deceleration is the Intelligent Driver Model (IDM), which can be used as a type of ACC model [24]. The effects of mechanical response are not included in the IDM and the maximum deceleration possible implies that brakes are activated. The time delay for effective brake activation, which can be an important factor, is not included in the model or in the present analysis.

The paper is organized as follows. Section 2 is about a simple model for which an analytic result is proven. Section 3 contains simulations illustrating the results of Section 2. Section 4 is devoted to avoiding collisions that the limits might cause. Section 5 reports simulations for the more general model where mechanical response is included. Section 6 pertains to non-equilibrium initial states that result in acceleration beyond the limits, yet cause no collisions. Conclusions are drawn in Section 7.

## 2. Simple model

In this section I consider a simple model that illustrates the consequences of imposing limits on the acceleration on an otherwise string-stable platoon. For simplicity, I take $\tau = t_d = 0$ (instantaneous mechanical response) and let the maximum acceleration and deceleration be $a_{\max}$. The maximum deceleration comes from the powertrain when the power is reduced rather than from braking. (Otherwise a braking algorithm treating the dynamics of activation must be included in the model.) All vehicles are taken to be identical and are labeled by $n = 1, 2 \ldots$ with $n = 1$ being the first vehicle of the platoon. (I take $n = 0$ to be the leading vehicle whose velocity profile is to be specified.) The equation of motion for vehicle $n$ is therefore

$$a_n(t) = A_n(t), \quad |A_n(t)| < a_{\max}, \tag{2.1a}$$

$$= a_{\max}\,\mathrm{sgn}\,[A_n(t)], \quad \text{otherwise}, \tag{2.1b}$$

with

$$A_n(t) = \alpha\left[V_{op}\left(x_{n-1}(t) - x_n(t) - D\right) - v_n(t)\right] + k\left[v_{n-1}(t) - v_n(t)\right], \tag{2.2}$$

where

$$V_{op}(z) = v_{\max}, \quad z \geq hv_{\max}, \tag{2.3a}$$

$$= \frac{z}{h}, \quad 0 < z < hv_{\max}, \tag{2.3b}$$

$$= 0, \quad z \leq 0. \tag{2.3c}$$

The unconstrained acceleration is $A_n(t)$ and $V_{op}(z)$ is the optimal velocity which is limited by a maximum velocity (*e.g.*, the speed limit) $v_{\max}$. Without the constraints on acceleration, the model is the same as the full velocity difference model of Ref. [25].

Next I calculate the acceleration of vehicle $n$ when vehicle $n - 1$ does not exceed the limits on acceleration. For simplicity I take $k = \frac{1}{h}$. At time $t = 0$ the system is in equilibrium. So for $n = 1, 2 \ldots$

$$v_n(0) = v_{n-1}(0), \tag{2.4a}$$

$$x_{n-1}(0) - x_n(0) - D = hv_n(0). \tag{2.4b}$$

Assume that

$$-a_{\max} \leq a_{n-1}(t) \leq a_{\max}. \tag{2.5}$$

Because the system is initially in equilibrium and $a_n(0) = 0$, the equation of motion is (assuming all velocities are positive, but less than $v_{\max}$)

$$\ddot{x}_n + \left(\alpha + \frac{1}{h}\right)\dot{x}_n + \frac{\alpha}{h}x_n = \frac{\alpha}{h}\left(x_{n-1} - D + \frac{1}{\alpha}v_{n-1}\right). \tag{2.6}$$

Eq. (2.6) remains valid until $|a_n(t)|$ exceeds $a_{\max}$ (if it ever does).

Let

$$y_n = \dot{x}_n + \alpha x_n, \tag{2.7}$$