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Multifractal analyses of daily rainfall time series in Pearl River basin of China

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HIGHLIGHTS

- Multifractal properties of daily rainfall data in Pearl river basin are examined.
- The universal multifractal model (UMM) and the MF-DFA are used.
- It is found that the empirical $K(q)$ curves can be fitted very well by UMM.
- The estimated values of H indicate that the data correspond to conserved fields.
- A relatively strong correlation between $K(2)$ and elevation is found.

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ABSTRACT

The multifractal properties of daily rainfall time series at the stations in Pearl River basin of China over periods of up to 45 years are examined using the universal multifractal approach based on the multiplicative cascade model and the multifractal detrended fluctuation analysis (MF-DFA). The results from these two kinds of multifractal analyses show that the daily rainfall time series in this basin have multifractal behavior in two different time scale ranges. It is found that the empirical multifractal moment function $K(q)$ of the daily rainfall time series can be fitted very well by the universal multifractal model (UMM). The estimated values of the conservation parameter H from UMM for these daily rainfall data are close to zero indicating that they correspond to conserved fields. After removing the seasonal trend in the rainfall data, the estimated values of the exponent $h(2)$ from MF-DFA indicate that the daily rainfall time series in Pearl River basin exhibit no long-term correlations. It is also found that $K(2)$ and elevation series are negatively correlated. It shows a relationship between topography and rainfall variability.

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1. Introduction

Rainfall is one of the most important variables studied because its non-homogeneous behavior in event and intensity, leading to drought, water runoff and soil erosion with negative environmental and social consequences [1,2]. Analysis and modeling of rainfall are significant research problems in applied hydro-meteorology [3]. Rainfall time series often exhibit strong variability in time and space.

Rainfall also exhibits scaling behavior in time and space (e.g. Refs. [3–7]). There is thus a need to characterize and model rainfall variability at a range of scales which goes beyond the scales that can be directly resolved from observations [8]. Investigation of the existence of fractal behavior in rainfall processes has been an active area of research for many years [9]. Some recent experiments have shown that scale invariance, in time and space, does exist in rainfall fields [10]. Olsson et al. [11] investigated the rainfall time series by calculating the box and correlation dimensions via a monodimensional fractal approach (simple scaling). Their results indicate scaling but with different dimensions for different time aggregation periods. Hence the investigated rainfall time series display a multidimensional fractal behavior. Venugopal et al. [12] employed the wavelet-based multifractal analysis to reexamine the scaling structure of rainfall over time. Molnar and Burlando [13] used the exponent of correlation function, a multifractal parameter, to study the seasonal and spatial variabilities. Using 2-dimensional Fourier series analysis and spectral analysis, Boni et al. [14] proposed a methodology to study the estimated index factor for rainfall in mountainous regions. During the past two decades, stochastic models of rainfall have increasingly exploited the property of multifractal scale invariance, resulting in multifractal models that are more advantageous over conventional models in rainfall representations [15–17].

The multiplicative cascade model has been widely used to study the multifractal properties of the rainfall data (e.g. Refs. [2,4–8,17–29]). Schertzer and Lovejoy [4] showed that statistically scaled invariant processes are stable and converge to some universal attractor, and thus can be defined by a small number of relevant parameters, specifically three with the universal multifractal framework.

The simple multifractal analysis (MFA) is based upon the standard partition function multifractal formalism [30], developed for the multifractal characterization of normalized, stationary measurements. Unfortunately, this standard formalism does not give correct results for non-stationary time series that are affected by trends or that cannot be normalized [31]. Thus, two generalizations of simple MFA were developed. One is the wavelet-based MFA which has been used to study rainfall data (e.g. Ref. [12]). Another generalization is the multifractal detrended fluctuation analysis (MF-DFA) [31] which is an extension of the standard detrended fluctuation analysis (DFA) introduced by Peng et al. [32,33]. DFA can be employed to detect long-range correlations in stationary and noisy nonstationary time series. It intends to avoid the unraveling of spurious correlations in time series. The DFA method has been successfully applied to problems in fields such as DNA and protein sequences (e.g. Refs. [32,34,35]) and hydrology (e.g. Refs. [36–40]). The MF-DFA is a modified version of DFA for the detection of multifractal properties of time series. It renders a reliable multifractal characterization of nonstationary time series encountered in phenomena such as those in geophysics [31,37,38,41–46]. The MF-DFA has also been successfully applied to problems in hydrology (e.g. Refs. [37–39]). The relationship between topography and rainfall variability is a very important issue in the study of rainfall.

Our work in this paper focuses on the multifractal properties of daily rainfall time series and possible relationships between the multifractal exponents and landscape properties. We use the universal multifractal model (UMM) proposed by Schertzer and Lovejoy [4] to fit the multifractal moment function $K(q)$ of the rainfall data and propose a method to estimate the parameters. We also adopt the MF-DFA approach to detect the correlation and multifractal properties of daily rainfall data in this paper.

As the largest watershed in South China, the Pearl River (Zhujiang in Chinese) delta is a composite drainage basin with a total area of 45.4×10^4 km^2, consisting of three major rivers (i.e., West River, North River, and East River) and several independent rivers in the downstream and delta regions (see Fig. 1). The Asian monsoon and moisture transport are the important influencing factors on precipitation patterns in this region. Given its large size and dominance of a sub-tropical humid monsoon climate, the Pearl River basin is under the influence of rainfall variability which is a highly complicated process in space and time. Zhang et al. [47] reported an increased high-intensity rainfall over the basin in conjunction with the decreased rainy days and low-intensity rainfall. It was also found that the abrupt changes of the precipitation totals (for annual, winter, and summer precipitation) occurred in the late 1970s, 1980s, and early 1990s, and the precipitation intensity basically increased after the change points [47,48]. In this paper, we study the daily rainfall data over the period from 1 January 1960 to 31 December 2005 at 41 locations in Pearl River basin using the UMM and MF-DFA methods. Parameters from the above MFAs are used to infer the spatial relationship of rainfall in Pearl River basin of China.

2. Multifractal analyses

2.1. Universal multifractal approach based on the multiplicative cascade model

Let $T(t)$ be a positive stationary stochastic process at a bounded interval of $\mathbf{R}$, assumed to be the unit interval (0, 1) for simplicity, with $E(T(t)) = 1$ (for a time series $x_i,\ i = 1, \ldots, L$, we can define $t_i = i/L$, and $T(t_i) = x_i/(\sum_{k=1}^{L} x_k)$). The smoothing of $T(t)$ at scale $r > 0$ is defined as $T_r(t) = \frac{1}{r}\int_{t-r/2}^{t+r/2} T(s)ds$. We consider the processes $X_r(t) = \frac{T_r(t)}{T_1(t)}$, $t \in [0, 1]$.