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Message survival and decision dynamics in a class of reactive complex systems subject to external fields

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HIGHLIGHTS

- The dynamics of decisions in complex networks with external fields are analyzed.
- Sufficient conditions for opinion extinction are presented.
- Analytic bounds for the effects of exogenous perturbations are established.
- Numerical simulations are presented for a power law degree distribution.

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ABSTRACT

In this study, the dynamics of decisions in complex networks subject to external fields are studied within a Markov process framework using nonlinear dynamical systems theory. A mathematical discrete-time model is derived using a set of basic assumptions regarding the convincement mechanisms associated with two competing opinions. The model is analyzed with respect to the multiplicity of critical points and the stability of extinction states. Sufficient conditions for extinction are derived in terms of the convincement probabilities and the maximum eigenvalues of the associated connectivity matrices. The influences of exogenous (e.g., mass media-based) effects on decision behavior are analyzed qualitatively. The current analysis predicts: (i) the presence of fixed-point multiplicity (with a maximum number of four different fixed points), multi-stability, and sensitivity with respect to the process parameters; and (ii) the bounded but significant impact of exogenous perturbations on the decision behavior. These predictions were verified using a set of numerical simulations based on a scale-free network topology.

1. Introduction

Recently, the problem of modeling, analyzing, and simulating rumor and decision dynamics in complex networks has attracted increasing attention, including studies of rumor spread in social and small-world networks [1], and decision dynamics in scale-free networks [2]. The problem of rumor spreading [1] considers the time evolution of rumor spreaders and stiflers, but the decision dynamics [2] are concerned with the time evolution of different decisions in terms of their associated competitive interplay, which is similar to that observed in biological studies of different predators competing for the same prey, chemical species competing for the same reactant, and a virus spreading via reinfection.

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The main background from related research areas is summarized next, without any claim of completeness.

1.1. Axelrod's cultural dissemination model and its extensions

Axelrod's seminal article on culture dissemination is included among previous studies that have provided inspiration for further research into opinion spreading dynamics [3]. In this study, the author proposed a model for the mechanism of how people become more similar in terms of their interaction and how this convergence stops before reaching completion. In his initial claim, Axelrod stated that agents which can interact and be influenced in this manner share common cultural attributes, whereas those that cannot interact retain their differences. The methodology utilized in Ref. [3] is based on the following three principles: (i) agent-based modeling, (ii) no central authority, and (iii) adaptive rather than rational agents. Axelrod [3] also defined the concept of a degree of cultural similarity among agents and simulated the dynamic process of social influence by repeating the following steps: (i) randomly select a site to be active and choose one of its neighbors, and (ii) these two sites interact with a probability that is equal to their cultural similarity. An interaction comprises the random selection of a feature where the active site and its neighbor differ (if there is a difference) and changing the active site's trait for this feature into the neighbor's trait for this feature.

It was observed in Ref. [3] that other factors such as the range of interaction affected the number of stable regions and this effect attracted our attention in a previous study [4,5], where we considered the emergence of nontrivial collective behavior. This study considered a lattice of locally interacting elements with an absorbing order–disorder phase transition and the behavior of this system under a field influence, which was based on the model of cultural dissemination proposed by Axelrod in Ref. [3]. In Ref. [4], exogenous and endogenous effects were integrated into the model proposed in Ref. [3] to produce a unified framework. In this framework, the agents can interact with their neighbors in the system and via mass media according to their cultural similarities, where their cultural attributes can be changed by the influence of culturally compatible neighbors as well as by their exposure to mass media. For further details see Refs. [4,5].

An attribute of the system is that it exhibits a phase because the system can be varied externally, e.g., its amplitude and frequency. We assume that B is uniform, i.e., the mass B. If this is the case, then the system can be estimated as $q_0 \approx 55$ in two dimensions [6] with a message intensity B_c and an order parameter g, where $g(B, q) \sim [B - k_2(\mathrm{e}^{-k_1 q} - \mathrm{e}^{-k_1 q_0})^{k_3(q_0-q)}]$.

1.2. Sociophysics and voter models

Another phenomenon related to spreading over networks that has attracted the interest of many researchers in the last decade is the propagation of political opinions in a social network [7–11]. In Ref. [7], the authors presented the results of a dual model of opinion networks, which complements agent-based opinion models by attaching a social agent (voters) network to a political opinion (party) network, which has its own intrinsic mechanisms of evolution. The evolution of the voter network involves adding and deleting links, where the changes in opinions are governed by social influences, political climate, attraction to a particular party, and interactions. For further details, see Ref. [7].

Another very interesting approach to the mathematical modeling of political opinions in a social network is that proposed in Ref. [8]. In Ref. [8], a modified version of a finite random field Ising ferromagnetic model in an external magnetic field at zero temperature was proposed for describing group decision making, where the fields could have a non-zero average. It was assumed that inter-individual conflicts were minimal. In these conditions, interactions led to group polarization. For further details, see Refs. [8–11].

1.3. Virus spreading in complex networks

The most notable differences in terms of these classic fields of competition dynamics are the social system dynamics in the topologies of the underlying contact networks. For example, social phenomena may involve different personalities where each has a different number of contacts, thus the dynamics have an intrinsic distributed character, where the emergence of nearly homogeneous groups (so-called clusters) is possible. These differences mean that it is necessary to analyze social behavior from the viewpoint of dynamic networks. It is particularly noteworthy that that the underlying network topology, unless it changes over time, always fulfills the same characteristics in terms of its node degree distribution (e.g., see Refs. [12–14]). Formal mathematical models can be developed and analyzed to understand the mechanisms that underlie the dynamic phenomena that occur in dynamic networks, as well as using numerical simulation studies.

In Ref. [2], the regularity of the spread of information and public opinions toward two competing products was analyzed in complex networks, with a particular emphasis on the dynamics of decision competition in scale-free networks. A simple linear model was proposed and simulated over time, which showed that in contrast to most previous models based on modified SIS and SIR models [15–23], various types of information frequently spread simultaneously (different viruses, multiple opinions, rumors, etc.) in real-life settings and these may be mutually strengthened or even annihilated, unlike the dynamics of single opinion spread. The competition-based dynamics of two opinions that flowed freely in a complex network were studied and some interesting characteristics of the behavior of decision competition were determined.

However, studying the underlying mechanisms of the mutual influence between nodes in a network is expected to lead to nonlinear dynamics, as found in the models based on SIS and SIR mentioned above. Thus, it may be assumed that inherent competition mechanisms lead to classical nonlinear phenomena such as multiple attractors and parameter sensitivity.