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Quantum correlations from classically correlated states

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HIGHLIGHTS

- We study quantum correlations arising out of classically correlated states.
- We systematically explore this effect for families of three qubits states.
- We uncover a relation between classical mutual information and the maximum quantum correlation of the reduced states.

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ABSTRACT

Consider a bipartite quantum system with at least one of its two components being itself a composite system. By tracing over part of one (or both) of these two subsystems it is possible to obtain a reduced (separable) state that exhibits quantum correlations even if the original state of the full system is endowed only with classical correlations. This effect, first pointed out by Li and Luo in (2008), is of considerable interest because there is a growing body of evidence suggesting that quantum correlations in non-entangled, mixed states may constitute a useful resource to implement non trivial information related tasks. Here we conduct a systematic exploration of the aforementioned effect for particular families of states of quantum systems of low dimensionality (three qubits states). In order to assess the non-classicality of the correlations of the reduced states we use an indicator of quantum correlations based upon the state disturbances generated by the measurement of local observables. We show, for a three-qubit system, that there exists a relationship between the classical mutual information of the original classically correlated states and the maximum quantum correlation exhibited by the reduced states.

1. Introduction

It has been realized in recent years that the quantum features of the correlations exhibited by multipartite quantum systems are manyfold, entanglement being only one of the possible non-classical manifestations [1–21]. Even separable mixed states (that are not entangled) can have correlations with subtle non-classical properties [1]. Several quantitative measures have been proposed to study the different non-classical aspects (besides entanglement) of the correlations appearing in composite quantum systems. Among these measures we can mention quantum discord [1] and the measures of correlations based on the disturbances of quantum states arising from local measurements. The latter ones were advanced by Luo [5–7] and by SaiToh and collaborators [14,15]. In the case of pure states all these measures coincide with quantum

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entanglement. However, in the case of mixed states these quantities correspond to physical properties of quantum systems that differ from entanglement.

It is generally agreed that the states of a bipartite system that are to be regarded as being only classically correlated (that is, having no quantum correlations) are those described by density matrices that are diagonal in a product basis $\{|i\rangle\, |j\rangle,\ i = 1, \dots, N_1;\ j = 1, \dots, N_2\}$, where $\{|i\rangle,\ i = 1, \dots, N_1\}$ and $\{|j\rangle,\ j = 1, \dots, N_2\}$ are orthonormal bases associated with the two subsystems, $N_{1,2}$ being the dimensions of the concomitant Hilbert spaces. It is worth stressing that the set of classical states is different from the set of separable (that is, non-entangled) states. Indeed, there are important differences between these two sets. For instance, the set of separable states is convex, while the set of classical states is not [7]. Also, measures of non-classicality such as discord do not satisfy monogamy relations [21], which constitute a basic property of quantum entanglement. It is usually assumed that classical states do not provide resources for information processing or information transmission, based on quantum correlations. Consider two parties A and B, that share a quantum state of a bipartite system consisting of two subsystems a and b (A is in possession of subsystem a while B is in possession of subsystem b). Now, assume that one or both subsystems (a and b) are themselves composite systems. Then, as it was shown by Li and Luo, tracing over part of one (or both) subsystems a and b, it is possible to obtain a state with quantum correlations, even if the original joint state of the composite ab was classical. This is an interesting effect, because it indicates that the aforementioned classical state of the composite system AB may have some "hidden" quantum correlations. The aim of the present contribution is to study in detail this effect for some families of states of systems of three qubits.

The approach to quantum correlations proposed by Luo [5] on the basis of measurement induced disturbances has two desirable features. First, it has a direct and intuitive interpretation in terms of the basic notion that in classical settings one can do a measurement on a system without disturbing it. In quantum scenarios, on the contrary, measurements usually lead to disturbances on the systems being measured. Luo applies these ideas to the study of correlations in bipartite systems. According to this approach, a bipartite system has only classical correlations if it is possible to conduct local measurements on both subsystems that do not disturb the global state of the composite system. If this cannot be done, the (minimum) size of the disturbance due to local measurements constitutes a quantitative measure of the quantumness of the correlations exhibited by the system under consideration. Another advantage of Luo's proposal is that the concomitant measure of the quantum character of correlations is computationally more tractable than other measures, such as quantum discord. It is important to emphasize that both quantum discord and the notion of quantum correlations based upon measurement induced disturbances determine the same family of classical states of a quantum bipartite system. As already mentioned, these states are those described by density matrices that are diagonal in a product basis. Indeed, it is shown in Ref. [5] that a quantum state ρ of a bipartite system is undisturbed by appropriate (un-read) local measurements if and only if ρ is diagonal in a product basis. This suggests a natural way of assessing the "amount of quantumness" exhibited by the correlations present in a quantum state ρ, by recourse to the minimum possible "distance" between ρ and the disturbed state $\Pi(\rho)$ resulting from a local measurement [5].

2. Non-classicality indicators based on measurement induced disturbances

Given a bipartite system's density matrix ρ^{ab}, with $\rho^a := \mathrm{tr}_b \rho^{ab}$ and $\rho^b := \mathrm{tr}_a \rho^{ab}$ the pertinent reduced densities, one defines the measure

$$\mathcal{M}(a, b) := \mathcal{I}(a, b) - I_C(a, b), \tag{1}$$

where $\mathcal{I}(a, b) := S(a) + S(b) - S(a, b)$ is the mutual quantum information between the two parties $a - b$ of ρ^{ab} and $I_C(a, b)$ is the classical mutual information ascribed to the post-measurement state $\Pi[\rho^{ab}] := \sum_{m,n} \Pi_{mn} \rho^{ab} \Pi_{mn}$, such that $\{\Pi_{mn}\} = \{\Pi^a_m \otimes \Pi^b_n\}$ is a projective measurement (complete and bi-local) on ρ^{ab}. $S(\rho) = -\mathrm{tr}(\rho \log \rho)$ is the von Neumann entropy of the state ρ.

We are particularly interested in Measurement Induced Disturbances (MIDs). Now the set $\{\Pi^a_m\}$ ($\{\Pi^b_n\}$) corresponds to the eigenprojectors of the spectral decomposition of the state ρ^a (ρ^b). Since MIDs do not involve any kind of optimization, they may overestimate sometimes quantum correlations. This problem has been dealt with in several ways. One of them is the symmetric discord

$$\mathcal{M}_{\mathcal{S}}(a, b) := \inf_{\{E_a \otimes E_b\}} \{\mathcal{I}(a, b) - I'(a, b)\}, \tag{2}$$

where $I'(a, b)$ corresponds to the classical mutual information of the post-measurement state resulting from the general local measure $\{E_a \otimes E_b\}$ [16,8,9].

Our main goal here is to detect non-classicality. As a consequence, overestimation does not constitute an important problem for us. Thus, we focus attention on MIDs given the tractability of the associated computational problem, both from the analytical and the numerical viewpoints.

As mentioned before, a bipartite state is *classical* if and only if it is diagonal in one special basis, that of the local eigen-projectors, and can thus be expressed as

$$\rho^{ab} = \sum_{m,n} p_{mn} \Pi^a_m \otimes \Pi^b_n, \tag{3}$$

with $\{p_{mn}\}$ a bivariate probability distribution $p_{mn} \geq 0$ and $\sum_{m,n} p_{mn} = 1$.