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Random Ising antiferromagnet on Bethe-like lattices with triangular loops

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HIGHLIGHTS

- Random Ising antiferromagnet on triangular cactus lattices is studied.
- Antiferromagnetic phase does not exist for smaller number of corner sharing triangles.
- Spin glass phase shows a reentrant behavior when there is no antiferromagnetic phase
- Spin glass phase in the antiferromagnet appears for weak randomness.

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ABSTRACT

Phase diagrams for a random Ising antiferromagnet on Bethe-like lattices with triangular loops are obtained. Triangular loops cause strong geometrical frustration for the Ising antiferromagnet. Spin glass states appear by introducing randomness in the interaction between Ising spins. The random Ising antiferromagnet is studied by the replica method using global order parameter. The phase diagrams are compared with those for the corresponding random Ising ferromagnet to see the effects of the geometrical frustration. Antiferromagnetic phase does not appear for $M \leq 4$ where M is the number of the corner sharing triangles on the Bethe-like lattices. In these cases, spin glass phase appears with a reentrant behavior. Spin glass phase in the random antiferromagnet appears for much weaker randomness than that in the corresponding random ferromagnet.

1. Introduction

Frustration and randomness are important for the appearance of the spin glass phase. For the Ising antiferromagnet, strong geometrical frustration exists in the models on lattices containing triangles with nearest neighbor antiferromagnetic interactions. The frustration suppresses long-range antiferromagnetic ordering. The models on the Kagomé lattice [1] and the triangular lattice [2] have been investigated. Randomly diluted Ising antiferromagnet on the triangular lattice has been investigated focusing on the glassy order [3,4].

Among approximations for the models with geometrically frustrated triangles, the models on Bethe-like lattices with 2 and 3 corner shearing triangles per site approximate the Kagomé lattice and the triangular lattice, respectively. The pure antiferromagnetic Ising model on the Bethe-like lattice with 2 corner shearing triangles in zero magnetic field does not have an ordered phase even at zero temperature [5], which agrees with the result for the corresponding model on the Kagomé lattice [6]. The model with 3 corner shearing triangles in zero magnetic field does not order [7]. This is in accord with the result for the model on the triangular lattice [8].

In this paper, Ising antiferromagnet with randomness in the exchange interaction on the Bethe-like lattices is studied. Phase diagrams are obtained and compared with those for the corresponding Ising ferromagnet to see the effects of the

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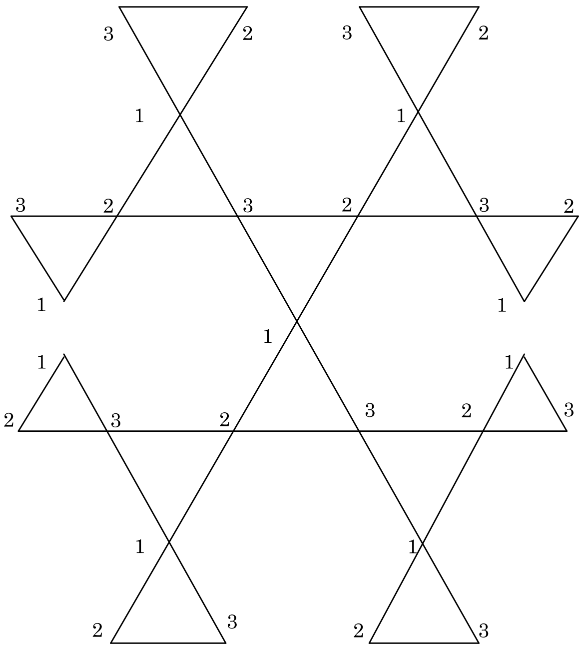

Fig. 1. A portion of a Bethe-like lattice with $M = 2$. The lattice is divided into three sublattices as numbered in the figure.

geometrical frustration. We study the random Ising models by the replica method. For $2 \leq M \leq 4$ where M is the number of corner shearing triangles per site, the Ising antiferromagnet does not show antiferromagnetic order. Spin glass phase appears with a reentrant behavior. For $M \geq 5$, antiferromagnetic phase appears as well as the spin glass phase. Spin glass phase appears for much weaker randomness compared with the case of the corresponding ferromagnetic models. For the antiferromagnet with $M = 4$, the spin glass phase appears as weak randomness as $J/(-J_0) \simeq 0.102$ where J_0 and J^2 are the mean and variance of the Gaussian random exchange interaction, respectively. On the other hand, much stronger randomness of $J/J_0 \gtrsim 2.463$ is needed for the corresponding ferromagnet.

In Section 2, the replica method using global order parameter [9–13] is sketched. Phase diagrams for the random Ising antiferromagnet are obtained and compared with those for the corresponding Ising ferromagnet in Section 3. Conclusions are given in Section 4.

2. Model and replica method using global order parameter

The Hamiltonian of the Ising antiferromagnet with random exchange interaction is given by

$$H = -\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j \tag{1}$$

where $\sigma_i = \pm 1$ and the summation is over all links. The quenched random exchange J_{ij} obeys a Gaussian probability with the mean value J_0 and variance J^2. For the antiferromagnet, $J_0 < 0$.

In this paper, the model is studied on Bethe-like lattices with triangular loops. A portion of a lattice with $M = 2$ is depicted in Fig. 1 as an example. The lattice is divided into three sublattices as shown in the figure.

The free energy F on the Bethe-like lattices can be represented as a summation of one body and cluster free energies [14]. The free energy is given by

$$F = \sum_i F_i + \sum_\mu \left(F_\mu - \sum_{(j)} F_{\mu(j)} \right) \tag{2}$$

where the indexes i and μ represent a site and a triangular cluster respectively, and (j) is the sublattice index. Introducing effective field distribution coming from triangle clusters connected at a sublattice (i) as $P(h_{(i)})$, the free energy per site f is given by

$$\beta f = \frac{M-1}{3} \sum_{(i)=1}^{3} \int_{-\infty}^{\infty} \mathrm{d}h_{(i)} P^{(M)}(h_{(i)}) \ln[\cosh(\beta h_{(i)})] - \frac{M}{3} \int \prod_{ij} \mathrm{d}J_{ij}\rho(J_{ij}) \prod_{(i)} \mathrm{d}h_{(i)\mu} P^{(M-1)}(h_{(i)\mu})$$
$$\times \ln \mathrm{Tr}_{\{\sigma_{(i)}\}} \exp\left(\sum_{ij} \beta J_{ij}\sigma_i\sigma_j + \beta \sum_{(i)} h_{(i)\mu}\sigma_{(i)} \right). \tag{3}$$