Physica A 447 (2016) 527–534

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractality of stock markets based on cumulative distribution function and multiscale multifractal analysis

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HIGHLIGHTS

- We analyze Chinese and US stock markets during the period of 1992 to 2012.
- The cumulative distribution function of modified Hurst surface is calculated.
- Stable structures of multifractal scaling are found by performing CDF-MMA technique.

ARTICLE INFO

Article history:
Received 28 July 2015
Received in revised form 29 October 2015
Available online 29 December 2015

Keywords:
CDF
Multiscale
MF-DFA
MMA
Stock market

ABSTRACT

Considering the diverse application of multifractal techniques in natural scientific disciplines, this work underscores the versatility of multiscale multifractal detrended fluctuation analysis (MMA) method to investigate artificial and real-world data sets. The modified MMA method based on cumulative distribution function is proposed with the objective of quantifying the scaling exponent and multifractality of nonstationary time series. It is demonstrated that our approach can provide a more stable and faithful description of multifractal properties in comprehensive range rather than fixing the window length and slide length. Our analyzes based on CDF-MMA method reveal significant differences in the multifractal characteristics in the temporal dynamics between US and Chinese stock markets, suggesting that these two stock markets might be regulated by very different mechanism. The CDF-MMA method is important for evidencing the stable and fine structure of multiscale and multifractal scaling behaviors and can be useful to deepen and broaden our understanding of scaling exponents and multifractal characteristics.

1. Introduction

Finance is an active research area, and a number of studies have used statistical mechanics to investigate the dynamics of financial markets due to the complex structures. Application of the idea gained from fractal theory to study the dynamical properties has been one of the most exciting areas of research in recent times [1–6]. Various complex systems have considered to be well represented by multifractality [7–21]. In previous study, it is widely accepted that financial market illustrates strong sign of complexity, power-law and multifractality [22–28]. The characteristics of the correlation in complex system can be investigated by extracting its scaling exponents. While some processes (monofractal) can be quantified by a single scaling exponent, others (multifractal) require the spectrum of exponents to characterize.

The detrended fluctuation analysis (DFA) invented by Peng et al. [12] has been established as an important tool for the determination of fractal scaling properties and detection of long-range power-law correlations in signals. Multifractal

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http://dx.doi.org/10.1016/j.physa.2015.12.012
0378-4371/

detrended fluctuation analysis (MF-DFA) method [29] is an universal tool to investigate multifractality, which is a multifractal generalization of the DFA method. In the previous studies, the MF-DFA exponents were estimated as the slope of a MF-DFA function plotted versus the resolution s on a log–log scale. The scaling range for determining the scaling exponents cannot be defined precisely, usually determined by visual inspection of plot. However, it is not adequate to describe the dynamical behaviors of time series by using single or two scaling exponents. Then Gierałtowski et al. [30] generated a new method called multiscale multifractal detrended fluctuation analysis (MMA) for solving the above problem. This new technology allows us to investigate not only the multifractal properties but also dependence of these properties on the time scale. MMA has successfully been applied to diverse field such as heart rate dynamics, economics and traffic systems [31–33].

Hence, previous studies on multiscale DFA inspire us to propose cumulative distribution function (CDF) statistics of Hurst surface $h(q, s)$ based on MMA method, which is called CDF-MMA method in our paper. In order to avoid the individual blind spot, the ensemble group averages of CDF values corresponding to individuals are calculated to explore the scaling behaviors. The present study shows more stable and robust results, allowing the assessment of multifractality and permitting individual discrimination.

The organization of this paper is as follows. First, the artificial and real-word data sets used in our work are given in Section 2. Section 3 presents methods employed in study. Section 4 is devoted to show the results by employing our modified CDF-MMA approach. Finally, the important conclusions drawn from this study are provided in Section 5.

2. Data

2.1. Artificial data

In order to test validity of the proposed method, we present results for several artificial series: random series, monofractal noise and binomial multifractal series. We now turn to the artificial processes.

(1) Monofractal noise

Many empirical data sets are characterized by long-range power-law auto-correlations. In the present study we generate artificial monofractal noise data sets of $N = 2^{15}$ samples by using modified Fourier filtering method [34] with scaling exponents $\alpha = 0.2$, $\alpha = 0.4$, $\alpha = 0.5$ and $\alpha = 0.7$ respectively. The artificial monofractal noises are plotted in Fig. 1.

(2) Binomial multifractal model

The artificial multifractal data sets considered in this paper are generated by applying binomial multifractal model [29]. In the binomial multifractal model, a series of $N = 2^{n_{\max}}$ numbers k with $k = 1, 2, \ldots, N$ is defined by

$$X(k) = \beta^{n(k-1)}(1-\beta)^{n_{\max}-n(k-1)} \tag{1}$$

where $0.5 < \beta < 1$ is a parameter and $n(k)$ is the number of digits equal to 1 in the binary representation of the index k, e.g. $n(13) = 3$, since 13 corresponds to binary 1101. In our study, we consider binomial multifractal series with parameters $\beta = [0.54, 0.56, 0.58, 0.6]$ and $n_{\max} = 15$. The time series generated by the binomial multifractal models with these parameters are shown in Fig. 2.

2.2. Real-world data

To show how the method applies to real data, we study different real-world financial series, which we consider the outputs of the complex systems-daily closing values of six stock indices. The data sets are obtained from Yahoo Finance covering 5533 days from May 12, 1992 to May 8, 2012. The original stock market indices are shown in Fig. 3. For simplicity, DJI, NYSE, SP500, HSI, ShangZheng and ShenCheng are used for representing six stock markets respectively. Looking at the closing indices of every day in which there is negotiation, we consider the normalized log-returns. Let $P_i(t)$ be the index of the stock market $i = 1, 2, \ldots, N$ at time t and $t = 0, 1, \ldots, T$. The absolute return is calculated as $R_i(t) = \ln(P_i(t)) - \ln(P_i(t-1))$.

3. Methodology

3.1. MF-DFA method

The multifractal detrended fluctuation analysis (MF-DFA) method was developed by Kantelhardt et al. [29] for the multifractal characterization of non-stationary time series. MF-DFA is a generalization of the detrended fluctuation analysis (DFA) method. MF-DFA method can be described as follows. Let us suppose that x_t is a series of length N, and this series is of compact support, i.e. $x_t = 0$ for an insignificant fraction of the values only. The corresponding profile $Y(i)$ is computed by integration as

$$Y(i) = \sum_{t=1}^{i}(x_t - \langle x\rangle), \quad i = 1, 2, \ldots, N. \tag{2}$$