Physica A 447 (2016) 100–107

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Correlated continuous time random walk and option pricing

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ARTICLE INFO

Article history:
Received 20 October 2015
Received in revised form 1 December 2015
Available online 15 December 2015

Keywords:
Anomalous diffusion
Correlated continuous time random walk
Option pricing
Transaction cost

ABSTRACT

In this paper, we study a correlated continuous time random walk (CCTRW) with averaged waiting time, whose probability density function (PDF) is proved to follow stretched Gaussian distribution. Then, we apply this process into option pricing problem. Supposing the price of the underlying is driven by this CCTRW, we find this model captures the subdiffusive characteristic of financial markets. By using the mean self-financing hedging strategy, we obtain the closed-form pricing formulas for a European option with and without transaction costs, respectively. At last, comparing the obtained model with the classical Black–Scholes model, we find the price obtained in this paper is higher than that obtained from the Black–Scholes model. A empirical analysis is also introduced to confirm the obtained results can fit the real data well.

1. Introduction

In physical world, the continuous time random walk (CTRW) model was originally introduced by Montroll and Weiss in 1965 [1]. Since that time, the CTRW was proved to be a useful tool to describe the anomalous diffusion phenomena in terms of the mean square displacement (MSD)

$$\langle X^2(t)\rangle \simeq K_\gamma t^\gamma, \quad (1)$$

here, the value $\gamma > 1$ characterizes a super-diffusive process, $\gamma < 1$ a sub-diffusive one, and $\gamma = 1$ a normal diffusion. Within the simplest CTRW picture, the diffusion of a particle is considered as a sequence of independent random jumps occurring instantaneously; the waiting time between the successive jumps is also independent random variables. Thus, the motion of a particle is completely determined by the two probability density functions, namely, jump length PDF and waiting time PDF. Normal distribution jump length and exponential distribution waiting time lead to the normal diffusion model; If the jump length is still normal distribution, but the waiting time is changed to power-law distribution, the diffusion model turns to anomalous, namely, the PDF satisfies to a fractional Fokker–Planck equation [2]. But, this independence of the waiting time and jump lengths gives rise to a renewal process is not always justified. As soon as the random walk has some form of memory, even a short one, the variable becomes non-independent. Examples are found in financial market dynamics [3], single trajectories in which there is a directional memory [4], or in astrophysics [5]. Recently, these facts impel one to introduce the correlated continuous time random walks (CCTRW) [6–9]. Some CCTRW models with long-range correlations are studied in Refs. [10,11].

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http://dx.doi.org/10.1016/j.physa.2015.12.013

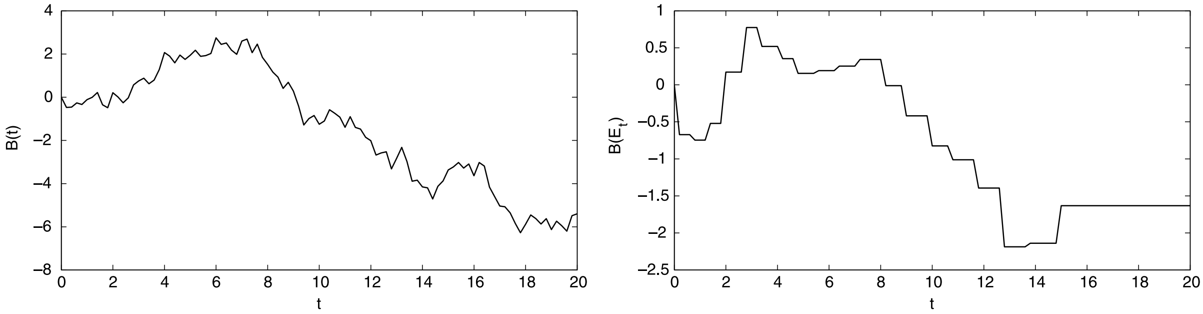

Fig. 1. Comparison of trajectory of the standard Brownian motion $B(t)$ (left) and the subordinated Brownian motion $B(E_t)$ (right) with $\alpha = 0.7$.

In financial market, the classical Black–Scholes model is based on the diffusion process called geometric Brownian motion [12,13]

$$dS_t = \mu S_t dt + \sigma S_t dB(t), \tag{2}$$

where S_t is the stock's price, and μ, σ are the expected average growth for the price and the expected noise intensity (the volatility) in the market dynamics, respectively. dS/S is usually called price return. The model is a geometric random walk with drift μ and diffusion σ. $B(\tau)$ is the standard Brownian motion.

However, on the one hand, the empirical studies show that many characteristic properties of markets cannot be captured by this model [14]. For example, the leptokurtic feature, in other words, the historical data shows that the return distribution has a higher peak and heavier tails than that of the normal one [15]. In Refs. [16,17] Bonanno applied a modified nonlinear Heston model to analyze the dynamics of stock prices with stochastic volatility.

On the other hand, in a complete financial market without transaction costs, the Black–Scholes no-arbitrage argument provides a hedging portfolio that replicates the option. However, the Black–Scholes hedging portfolio requires continuous trading and therefore, in a market with propositional transaction costs, it is expensive. Leland [18] first examined option replication in the presence of transaction costs (TC) in a discrete time setting, and poses a modified replicating strategy, which depends upon the level of transactions costs and upon the revision interval, as well as upon the option to be replicated and the environment. Since then, a lot of authors study this problem, all in a discrete time setting [19–26]. Recently, Li [27] introduced the Levy Jump into the option pricing with transaction costs.

In the paper [28,29], researchers applied continuous-time random walks in finance and economics. Magdziarz applied the CTRW into the option pricing problem [30]. He introduced the subdiffusive geometric Brownian motion (SGBM) as the model of asset prices, which means the time t in the right side of Eq. (2) is changed into the inverse of a α-stable *Lévy* motion. Magdziarz showed that the considered model is arbitrage-free but incomplete, since the equivalent martingale measure is not unique. But, under a special equivalent martingale measure, they obtained the corresponding subdiffusive BS formula for the fair prices of European options. In Refs. [31,32], Magdziarz extended the α-stable *Lévy* motion to a more general *Lévy* motion, and the corresponding BS formula is also obtained.

Since, in real financial market, the expected return is linearly correlative to time and the leptokurtic feature exits, in this paper, we introduce the correlated CTRW into the option pricing problem and suppose that the underlying of the option contract is driven by a subordinated geometric Brownian motion, therefore, the price of underlying S_t is given as

$$dS_t = \mu S_t dt + \sigma S_t dB(E_t), \tag{3}$$

where, E_t is the inverse of

$$A(\tau) = \frac{1}{\tau}\int_0^\tau U_\alpha(\xi)d\xi. \tag{4}$$

In mathematical form, E_t can be expressed as

$$E(t) = \inf\{\tau \geq 0 : A_\tau > t\}, \tag{5}$$

where $U_\alpha(\xi)$ is the strictly increasing α-stable Lévy motion with Laplace transform given by $E(e^{-kU_\alpha(\xi)}) = e^{-\xi k^\alpha}$, $0 < \alpha < 1$, and E_t is independent of $B(\tau)$. Since the every jump length of $U_\alpha(\xi)$ is not the same, then the period of its inverse time is not the same [30]. Here, $A(\tau)$ is the average of $U_\alpha(\xi)$ over the time, which leads to the same period of inverse time, see Fig. 1. So, for every jump of $U_\alpha(\xi)$, there is a corresponding flat period of its time averaged inverse $E(t)$. These heavy-tailed flat periods of $E(t)$ represent long waiting times in which the subdiffusive particle gets immobilized in the trap, and the every immobilized time is the same.

The paper is organized as follows. In Section 2, we prove that the distribution of the return of the underlying is heavy-tailed and thin peaked when the underlying price follows Eq. (3). In Section 3, the evaluation formula of a European option without transaction costs is derived. Then, we investigate the option pricing under transaction costs in discrete time, and a closed form representation of the option pricing formula is given. A empirical analysis is also made to prove the validity of these results. In Section 4, we present our conclusions.