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Dynamical analysis for a model of asset prices with two delays ☆

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HIGHLIGHTS

- Bifurcation analysis is employed to study the market stability or oscillation.
- Use center manifold reduction method to investigate the stability and criticality.
- Delay induces supercritical bifurcation and makes oscillating period increase.

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ABSTRACT

This paper provides a new perspective to understand the mechanism on the market stability or oscillation by investigating a two-dimensional asset price model with two delays. Stability conditions and the existence of Hopf bifurcation are obtained by investigating the characteristic equation. Then an explicit algorithm for determining the criticality of Hopf bifurcation and stability of the bifurcating solutions is derived, using the center manifold reduction method. The global continuation of bifurcating periodic solutions is detected using a global Hopf bifurcation theorem. It is found that delay may induce supercritical Hopf bifurcations, hence bring oscillation into the asset price model. Moreover, when time delay gets larger, the period of oscillation also increases. Finally, some numerical illustrations with Matlab and DDE-Biftool are carried out to support the theoretical analysis.

1. Introduction

Asset Pricing Theory is a very popular research topic in the field of financial economics. The value at risk model (VaR model) and the central model of finance (CAPM model) are widely used in the dynamic study of financial market [1]. In the above models, the relationship between the assets expected rate of return and risk assets in the securities market is illuminated. But they cannot explain some stylized facts such as peak, fat tail and long term memory in financial market. People expect to get information from the financial market to make predictions about the future. Market fluctuation can change the supply and demand. Through the past prices and other transaction data, technical analysts use various technical trading methods to predict the price movements. In recent years, many scholars proposed financial models have different beliefs and behavior of the trader's (Heterogeneous Agent Model) to explain the stylized facts. Such theoretical models have been developed to explain phenomena such as market cycles, speculative bubbles and crashes [2,3].

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Most empirical researches showed that financial market was a complicated nonlinear system. It is efficient to study asset price theory by establishing the differential equation model based on HAM to describe the wide price fluctuation in the financial market. Extensive literatures introduced how to exploit delay differential equations to describe commodity prices and the cyclical economic behavior. Furthermore, studying solutions and their dynamical behaviors of models involving time delays is attractive. Some heterogeneous phenomena in financial market can be illuminated through stability analysis and bifurcation analysis of the model.

In 2005, Dibeh [4] presented a time-delay model to describe the effect of time delays on the dynamics of asset prices. However, asset cross-correlations in financial market have been found by theoretical and empirical analyzing [5–7]. In reality, multiple assets trade in a financial market. Asset prices exhibit a certain correlation due to public factors such as interest rates, exchange rate and traders. The stylized facts cannot be ignored, they can lead to the contagion of prices dynamics across assets in financial markets. When an asset market showed a trend of fluctuations, due to the effect of the parameters, the price fluctuations may transfer from one market to another market. In the financial crises, asset correlations have been shown to be responsible for large market fluctuations. For example, correlation between stocks increases in economic crisis, then stocks influence each other.

Most theoretical models that are used to explain the dynamics of such contagion phenomena are mainly probabilistic models [8,9]. To display correlation of asset prices how to affect volatility transmission in financial market clearly, Dibeh [10] proposed a 2-dimensional delay differential equation based on the model presented in Ref. [4] with coupling between the two markets and time delays in reaction times. In Ref. [10], the market is divided into two types of agents: the fundamentalists and chartists. The chartists make their decisions based on the information of past prices, which makes usually the model a delay differential system. The fundamentalists decide to buy or sell the asset through evaluating the difference between the actual price of the asset and the fundamental price of the asset. The model is given by

$$\begin{aligned}\frac{\mathrm{d}P_1(t)}{\mathrm{d}t} &= (1-m)\tanh(P_1(t)-P_1(t-\tau_1))P_1(t) - m(P_1(t)-v_1)P_1(t) + h(P_1(t)-P_2(t)),\\ \frac{\mathrm{d}P_2(t)}{\mathrm{d}t} &= (1-n)\tanh(P_2(t)-P_2(t-\tau_2))P_2(t) - n(P_2(t)-v_2)P_2(t) + \eta(P_2(t)-P_1(t)),\end{aligned} \tag{1}$$

where h and η represent the coupling strengths between the two markets. m and n represent fundamentalists' share of wealth in two markets, respectively. $1-m$ and $1-n$ represent chartists' share of wealth in two markets, respectively. To avoid being too general, we suppose $m \in (0, 1)$ and $n \in (0, 1)$. In Ref. [10], Dibeh showed that coupled speculative markets with different dynamics could influence each other through diffusive coupling by numerical simulation. He focused on the effects of feedback mechanisms. Extensive literatures indicate that delays can have complicated impact on the dynamics of asset prices system [11–14]. It is interesting to investigate how the time delays in the trend estimation equation affect attractor types. The purpose of this paper is to carry out a dynamic properties analysis of the system (1). We analyze not only the existence and stability of equilibrium point, but also the existence of the local Hopf bifurcations. We obtain the explicit algorithm for determining the direction of the Hopf bifurcations and the stability of the bifurcating periodic solutions. Furthermore, the global existence of periodic solutions is established.

The rest of this paper is organized as follows. In Section 2, we analyze the existence of positive equilibrium of the model, then we study the stability of positive equilibrium and obtain the sufficient conditions of the existence of local Hopf bifurcation. In Section 3, using the center manifold theorem and the normal form theory [15], the explicit algorithm for determining the direction of the Hopf bifurcations and the stability of the bifurcating periodic solutions is derived. In Section 4, the global existence of periodic solutions is established using a global Hopf bifurcation theorem given by Wu [16] in the case of $\tau_1 = \tau_2 = \tau$. In Section 5, we carry out some numerical simulations to illustrate the analysis above.

2. Stability analysis of the interior equilibrium

In this section, we study the existence and the stability of the positive equilibrium of system (1).

2.1. The existence of positive equilibrium

Denote by $f(P_1) = I_1{P_1}^3 - I_2{P_1}^2 + I_3P_1 - I_4$, with $I_1 = nm^2$, $I_2 = 2nm(h+mv_1)$, $I_3 = h^2n + 2nmv_1h + nm^2{v_1}^2 + nmv_2h + hm\eta$, $I_4 = nh^2v_2 + nmhv_1v_2 + hm\eta v_1$.

Lemma 2.1. *System (1) has positive equilibrium if and only if $f(P_1) = 0$ has root on $(0, \frac{h+mv_1}{m})$.*

Proof. Considering the definition of positive equilibrium, we get

$$\begin{cases} P_2 = P_1\left[1 - \dfrac{m(P_1-v_1)}{h}\right], \\ f(P_1) = 0. \end{cases} \tag{2}$$