Available online at www.sciencedirect.com
SCIENCE DIRECT
Physica A 367 (2006) 145–157
PHYSICA A
www.elsevier.com/locate/physa

Chaos synchronization in a lattice of piecewise linear maps with regular and random couplings

A.M. dos Santos[a,*], R.L. Viana[a], S.R. Lopes[a], S.E. de S. Pinto[b], A.M. Batista[b]

[a]*Departamento de Física, Universidade Federal do Paraná 81531-990, Curitiba, Paraná, Brazil*
[b]*Departamento de Matemática e Estatística, Universidade Estadual de Ponta Grossa 84033-240, Ponta Grossa, Paraná, Brazil*

Received 5 September 2005; received in revised form 11 November 2005
Available online 9 December 2005

Abstract

We investigate aspects of the spatio-temporal dynamics exhibited by a one-dimensional lattice of chaotic piecewise linear maps in a coupling prescription which includes both regular (nearest and next-to-nearest neighbors) and randomly chosen couplings. We discuss the conditions for the existence of chaotic synchronized states, and relate them to the coupling parameters. The transition to synchronized behavior is described, and we explore some statistical properties of the time it takes to achieve this regime.

Keywords: Synchronization of chaos; Small world; Coupled map lattices

1. Introduction

Coupled map lattices have been widely recognized as useful models for spatially extended dynamical systems. They present discretized space and time, whereas retaining a continuous state variable whose evolution is governed by a map [1]. In this way coupled map lattices have advantages over cellular automata, in that the former are able to generate local information and a rich spatio-temporal dynamics [2]. Most research done on coupled map lattices has focused on limiting cases of coupling: the so-called local, or Laplacian case takes into account the effects of only the nearest neighbors of a given lattice site [3]; whereas in the global case each map interacts with the "mean-field" generated by all lattice sites [4]. Recently, a form of coupling was proposed which considers the effect of all neighbors decreasing as a power-law with the lattice distance, and which reduces to the above mentioned coupling prescriptions as limiting cases [5].

Another classification of these systems comprises regular lattices, for which there is a kind of translational symmetry of the coupling term; and random lattices, which exhibits shortcuts between sites (not necessarily close to each other) randomly chosen according to a specified probability distribution. Recent investigations in lattice models of sociological interest, like the so-called small-world phenomenon, have raised the need of lattices which share properties of both regular and random lattices [6]. In regular lattices, the

*Corresponding author.
E-mail address: ansantos@fisica.ufpr.br (A.M. dos Santos).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2005.11.012

existence of interactions with the nearest neighbors of a given site leads to a modeling difficulty, in that the transmission velocity of signals is limited by the diffusive nature of the coupling. By way of contrast, many interesting models require the introduction of shortcuts for rapid signal propagation (like in epidemiological models for spread of infections or in models for propagation of gossips in social networks) [7].

Purely random lattices present a very small amount of clustering, which can be described as the property that, if two given sites are connected to a third one, hence these sites are also connected to each other. Some amount of clustering is highly desirable in models of social and epidemiological networks. On the other hand, regular lattices typically show a high level of clustering.

The quest for lattices with small average distance between sites, yet with a reasonably large degree of clustering has led to the introduction of models with both regular and random interactions [6,7]. The properties of such small-world lattices have been studied with respect to some aspects of their spatio-temporal dynamics, with special emphasis on synchronization [8]. The synchronization properties of any lattice are enhanced with the addition of the randomly chosen shortcuts, which is revealed by a variety of numerical diagnostics. Chaos synchronization is particularly interesting to investigate, since it depends in an intricate fashion on the dynamical properties of the coupled map lattice. The existence of chaos synchronization of small-world networks has been shown for a general class of such lattices [9–11].

The dynamical properties of the synchronized state in lattices with random nonlocal connectivity were investigated by Chaté and Manneville [12] and Gade [13]. In this paper, we aim to focus on a hitherto not completely understood issue, i.e., the dependence of the chaos synchronization of the lattice on the coupling parameters, when both local and nonlocal connections are present. The transition from non-synchronized to synchronized behavior is investigated by means of an order parameter that presents a sharp transition curve, relating the critical coupling strength for achieving synchronization with the probability of random shortcuts. Moreover, we investigate the time it takes to achieve a chaotic synchronized state, which is also a useful measure of how the introduction of random couplings helps to improve synchronization in the lattice system. We remark that the introduction of long range shortcuts is not the only way to induce chaotic synchronization in a locally coupled lattice: it can also be done by the introduction of a time delay in the couplings [14], and the addition of an external forcing [15].

This paper is organized as follows: in Section 2, we introduce a lattice with regular and random couplings and investigate the characterization of spatial patterns, focusing on synchronization of chaotic maps and its numerical detection. In Section 3, we investigate some dynamical properties of the synchronized state and their dependence with the coupling parameters. The numerical results are explained from the stability properties in the directions transversal to the synchronized manifold in the system phase space. Our conclusions are left to the last section.

2. Lattice spatio-temporal dynamics

Let us consider a lattice of N maps, each of them with its state variable at discrete time $n : x_n^{(i)}$, where $i = 1, 2, \ldots N$. A model presenting regular as well as random couplings is

$$x_{n+1}^{(i)} = (1-\varepsilon)f(x_n^{(i)}) + \frac{\varepsilon}{4+M}\left[f(x_n^{(i-1)}) + f(x_n^{(i-2)}) + f(x_n^{(i+1)}) + f(x_n^{(i+2)}) + \sum_{j=1}^{N} f(x_n^{(j)}) I_{ij} \right], \tag{1}$$

where $\varepsilon > 0$ is the coupling strength, and the local dynamics at each site is represented by a piecewise linear map $x \mapsto f(x) = \beta x \pmod 1$. For $\beta > 1$ the map displays strong chaos, with Lyapunov exponent $\lambda_U = \ln \beta$ (from now on we choose $\beta = 3$).

This coupling scheme has two contributions: a regular one, from the nearest and second-nearest neighbors to a given site; and a random term represented by the matrix elements I_{ij}. We introduce a fixed number M of randomly chosen shortcuts for each site, with uniform probability $p = M/N$. Hence, in each row of the $N \times N$ connectivity matrix I_{ij} there are M randomly-chosen entries equal to 1, the $N - M$ others being padded with