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Size dependence of effective mass in granular columns

Jia Liu [a], Qingfan Shi [a,*], Xuanwen Liang [a], Lei Yang [a,b], Gang Sun [b]

[a] *Department of Physics, Beijing Institute of Technology, Beijing 100081, China*
[b] *Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China*

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ABSTRACT

An experimental setup is developed to measure accurately and reproducibly the effective mass as well as its fluctuations at the bottom of the granular columns. It is found that the saturated mass M_s obeys a third power function of the diameter of the silos. The data obtained are in agreement with the modified Janssen model proposed by Vanel and Clement [L. Vanel, E. Clement, Eur. Phys. J. B 11 (1999) 525–533].

1. Introduction

Granular materials exhibit complex nonlinear behavior. Many statistical models and approaches are generally used to resolve these problems. The study of static stresses in granular assemblies has made great progress both theoretically and experimentally in recent years [1–10]. As one of the important research fields, the sensitivity of the static pressure at the bottom of a granular column has been thoroughly investigated by Vanel et al. [8–10]. They measured the fluctuations and variations of the mean pressure with a precise and reproducible method, and evidenced that the form of the saturation curve was strongly dependent on the global packing fraction, filling procedures and measuring methods. However, the mechanical description of static collective granular systems is still far from being thoroughly revealed and especially, more experimental works on the relationship between the saturation of the filling mass and the diameter of the silos, with obvious engineering implications, should be performed. The achievements of Vanel et al. inspire us to continue exploring the open problems in this field. The prediction of third power function between the saturated effective mass M_s and the diameter of column in the Janssen model (JM) motivates us to do experiments for testing the size dependence of effective mass in granular column system.

Furthermore, Vanel and Clement proposed a modified Janssen model (MJM), which fits better with the data points than the classical Janssen model [11]. It is an inevitable result because there are two parameters in MJM, while only one parameter exists in JM. So, more experimental works are needed to argue the physical meaning of the two introduced parameters in MJM.

In this paper, we report the experimental results of the size dependence of effective mass M_e in granular columns. We focus on the dependence of the saturated effective mass M_s on the diameter of column D and the rationality of MJM. We expect our experiment results to be helpful in understanding the nonlinear character of the force network in silos.

* Corresponding author.
E-mail addresses: qfshi123@bit.edu.cn (Q. Shi), gsun@aphy.iphy.ac.cn (G. Sun).

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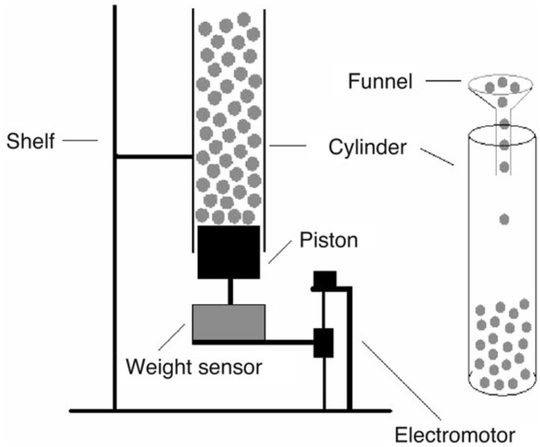

Fig. 1. The sketch map of the experimental setup and our filling means.

2. Experiments

A sketch of the experimental setup is shown in Fig. 1. A mass M of grains is poured into a vertical silo. The bottom of the silo is a movable piston which is designed so that neither touch the boundary nor leak the loaded grains. The moving speed of the piston is controlled by a high-performance motion electric machinery. The average pressure of the grains on the piston is entirely transmitted to a weight sensor which is connected to a computer. A brief relaxation time (normally 30–60 s) is allowed after the grains are slowly filled via a funnel into the cylindrical column. In this way, the kinetic energy of the initial falling grains will be relaxed. In order to get a set of reproducible results within a rather small fluctuation scale, the piston descends at a slow velocity (typically 0.05 mm/s) on a total distance of 20 mm. The slow downward motion of the piston could fully mobilize the frictional force between the granular column and the wall. Simultaneously, the average packing fraction tend to reach a stable value. Throughout the experiments, the total height of granular column is measured when its motion ceases so that the packing fraction can be calculated. In our case, the packing fraction is 0.590 ± 0.005.

The static pressure measured by the weight sensor at the bottom of silo is referred to as the effective mass M_t. As shown in Fig. 2, a set of measuring process is recorded, and M_t is observed to behave a big fluctuation at the beginning of measuring, but it becomes smaller with the descent displacement step and finally it becomes a rather small fluctuation scale. Here, we have taken the statistical average of 500 values of M_t in the final phase as depicted in the inset of Fig. 2 as a measuring value of M_e or M_s, while every data point shown in the following figures is the average of 5 measuring values.

As the filling mass increases, a saturation of the effective mass could be observed. In order to eliminate the effect of temperature drift in the process of measuring static pressure, the apparatus is settled in the surroundings of constant temperature. In this work seven silos of different diameter and grains of 2.65 mm diameter are used.

3. Rationality of MJM

Now let us investigate the effective mass as a function of the filling mass in the silo. As a pioneer in mechanical analysis of granular material, Janssen proposed a famous model to describe the pressure at the bottom of a container. In the classical Janssen model, the relationship between the filling mass M_f and the effective mass M_e is the following:

$$M_e = M_s(1 - e^{-M_f/M_s}), \tag{1}$$

where $M_s = \rho\pi(D/2)^3/(2\mu K)$, $M_f = \rho\pi(D/2)^2H$. ρ is the density of the material, μ is the coefficient of friction between grains and wall, H is the height of the grain column, and K is the ratio between horizontal and vertical stresses.

In the equation, the saturation mass is proportional to third power of the diameter of column. We test this law by pouring different mass of granular into the silos of different diameter. The evolution of the effective mass with the filling mass is shown in Fig. 3. Each data line represents silo's inner diameters 25.4 mm, 29.5 mm, 33.1 mm, 39.4 mm, 43.7mm, 46.3 mm, 50 mm, when counted from bottom to top respectively, and each data point has an error of less than 5%. We fit all the data points by JM within experimental precision, and get the saturated effective masses which are 18.1 g, 26.3 g, 40.8 g, 62.4 g, 84.6 g, 95.0 g, 128.0 g.

The dependence of saturated effective mass M_s on the diameter of the silos D is displayed in Fig. 4. It appears that the saturated effective mass changes nonlinearly with the diameter of the silos D. The relation of M_s and D is re-plotted in Log–Log format in the inset, which turns out to be linear and implies a power law which is 3.00 ± 0.10. This conclusion is in accordance with the Janssen model.