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Soliton solutions to the 3rd nonisospectral AKNS system

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Abstract

Bilinear form of the nonisospectral AKNS equation is given. The N-soliton solutions and double Wronskian solution are obtained through Hirota's direct method and Wronskian technique, respectively. The nonisospectral MKdV equation and its multi-soliton solutions are presented by reducing.

Keywords: Nonisospectral AKNS equation; Nonisospectral MKdV equation; Wronskian technique

1. Introduction

The nonisospectral AKNS system is an important physical models in $(1+1)$-dimensions because some of them can describe the waves in a certain type of nonuniform media [1–3]. These solutions of the nonisospectral AKNS system is a kind of dissipative waves. Its amplitude will diminish rapidly with the time increasing and its transmitting-direction will alter with the change of velocity. Up to now, there has been many methods for finding the exact solutions. For example, the nonisospectral KdV hierarchy has been investigated systematically through Darboux transformation [4] and some nonisospectral equations have been solved by using the IST [2,3,5]. In this paper, we will derive the novel solutions and double Wronskian solution of the nonisospectral AKNS equation by the Hirota's direct method [6] and Wronskian technique [7–9], respectively. We will also consider the reduction. By an easy transformation, we can get the nonisospectral MKdV equation, its N-soliton solutions and double Wronskian solution are given further.

The paper is organized as follows. In Section 2, the bilinear form of the nonisospectral AKNS equation is given and the N-soliton solutions are obtained. In Section 3, the double Wronskian solution is given. In Section 4, reducing the nonisospectral AKNS equation to nonisospectral MKdV equation by an easy transformation and giving the corresponding N-soliton solutions and double Wronskian solution. A conclusion and some remarks are given in the final section.

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2. Bilinear form and N-soliton solutions

Considering the nonisospectral AKNS equation

$$q_t = x(q_{xxx} - 6qrq_x) + 3(q_{xx} - 2rq^2) - 2q_x\partial^{-1}qr + 4q\partial^{-1}qr_x, \tag{1a}$$

$$r_t = x(r_{xxx} - 6qrr_x) + 3(r_{xx} - 2qr^2) - 2r_x\partial^{-1}qr + 4r\partial^{-1}rq_x \tag{1b}$$

with the Lax pair

$$\varphi_x = M(v,\eta)\varphi, \quad M(v,\eta) = \begin{pmatrix} -\eta & q \\ r & \eta \end{pmatrix}, \tag{2a}$$

$$\varphi_t = N(v,\eta)\varphi, \quad N(v,\eta) = \begin{pmatrix} A & B \\ C & -A \end{pmatrix}, \tag{2b}$$

where

$$A = -4\eta^3 x + 2\eta(xqr + \partial^{-1}qr) + xqr_x - xq_x r + \partial^{-1}(qr_x - rq_x),$$

$$B = 4xq\eta^2 - 2(q + xq_x)\eta + 2q_x + xq_{xx} - 2xq^2 r - 2q\partial^{-1}qr,$$

$$C = 4xr\eta^2 + 2(r + xr_x)\eta + 2r_x + xr_{xx} - 2xr^2 q - 2r\partial^{-1}qr,$$

q and r are potential functions, η is a spectral parameter, $\partial = \partial/\partial x$, $\partial\partial^{-1} = \partial^{-1}\partial = 1$.

This equation appeared in Refs. [10,11], where it is used to generate nonisospectral symmetries for the isospectral AKNS hierarchy. In this section, we will derive the bilinear form of the equation and obtain the N-soliton solutions through the Hirota's method.

Through the dependent variable transformation

$$q = \frac{g}{f}, \quad r = \frac{h}{f}. \tag{3}$$

Eq. (1) can be transformed into the bilinear form

$$D_t g \cdot f = (xD_x^3 + 3D_x^2)g \cdot f + 2D_x g \cdot f_x + 2gs, \tag{4a}$$

$$D_t h \cdot f = (xD_x^3 + 3D_x^2)h \cdot f + 2D_x h \cdot f_x - 2hs, \tag{4b}$$

$$D_x^2 f \cdot f = -2gh, \tag{4c}$$

$$D_x h \cdot g = D_x s \cdot f, \tag{4d}$$

where D is the well-known Hirota bilinear operator defined as

$$D_t^m D_x^n f \cdot g = (\partial_t - \partial_{t'})^m (\partial_x - \partial_{x'})^n f(t,x) g(t',x')|_{t'=t,\, x'=x}$$

and s is an auxiliary function. Substituting the expansion

$$f(t,x) = 1 + f^{(2)}\varepsilon^2 + f^{(4)}\varepsilon^4 + \cdots + f^{(2j)}\varepsilon^{2j} + \cdots, \tag{5a}$$

$$g(t,x) = g^{(1)}\varepsilon + g^{(3)}\varepsilon^3 + \cdots + g^{(2j+1)}\varepsilon^{2j+1} + \cdots, \tag{5b}$$

$$h(t,x) = h^{(1)}\varepsilon + h^{(3)}\varepsilon^3 + \cdots + h^{(2j+1)}\varepsilon^{2j+1} + \cdots, \tag{5c}$$

$$s(t,x) = 1 + s^{(2)}\varepsilon^2 + s^{(4)}\varepsilon^4 + \cdots + s^{(2j)}\varepsilon^{2j} + \cdots \tag{5d}$$

into Eq. (4) and comparing the coefficient of the same power of ε, we have

$$g_t^{(1)} = xg_{xxx}^{(1)} + 3g_{xx}^{(1)} + 2g^{(1)}, \tag{6a}$$