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Multicritical behavior of the antiferromagnetic spin-$\frac{3}{2}$ Blume–Emery–Griffiths model

Mustafa Keskin[a,*], M. Ali Pınar[b], Ahmet Erdinç[a], Osman Canko[a]

[a]*Department of Physics, Erciyes University, 38039 Kayseri, Turkey*
[b]*Institute of Science, Erciyes University, 38039 Kayseri, Turkey*

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Abstract

The multicritical behavior of the antiferromagnetic spin-$\frac{3}{2}$ Blume–Emery–Griffiths model in an external magnetic field is studied within the lowest approximation of the cluster variation method. We have investigated the thermal variations of order parameters for different values of interaction parameters and external magnetic field and constructed the resulting phase diagrams. The model exhibits distinct critical regions, including the first-order, second-order tricritical point, double critical point and zero critical point. Comparison of the phase diagrams with closely related systems is made.

Keywords: Spin-$\frac{3}{2}$ Blume–Emery–Griffiths model; Cluster variation method; Phase diagrams

1. Introduction

The spin-$\frac{3}{2}$ Blume–Emery–Griffiths (BEG) model is described by the Hamiltonian in an external magnetic field

$$\mathscr{H} = -J\sum_{\langle i,j\rangle} S_iS_j - K\sum_{\langle i,j\rangle} S_i^2S_j^2 + D\left(\sum_i S_i^2 + \sum_j S_j^2\right) - H\left(\sum_i S_i + \sum_j S_j\right), \tag{1}$$

where the spins S_i located at site i on a discrete lattice can take the values $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$, and the first two summations run over all nearest-neighbor pair of sites. J, K, D and H describe the bilinear interaction, biquadratic interaction, single-ion anisotropy and effect of an external magnetic field, respectively. The Hamiltonian and phase diagrams are invariant under the transformations ($H \to -H$ and $S \to -S$). The spin-$\frac{3}{2}$ BEG model is the most general spin-$\frac{3}{2}$ Ising model. The spin-$\frac{3}{2}$ Ising model Hamiltonian with only J and D interaction is known as the spin-$\frac{3}{2}$ Blume–Capel (BC) model, and the spin-$\frac{3}{2}$ Ising model Hamiltonian with only J and K interaction is known as the isotropic spin-$\frac{3}{2}$ BEG model.

The critical properties of the spin-$\frac{3}{2}$ BEG model for $K/J \geqslant 0$ have been studied and its phase diagrams have been calculated by renormalization-group (RG) techniques [1], the effective field theory (EFT) [2], the Monte

*Corresponding author. Tel.: +90 352 4374901x33105; fax: +90 352 4374933.
E-mail address: keskin@erciyes.edu.tr (M. Keskin).

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Carlo (MC) and a density-matrix-RG method [3]. An exact formulation of the spin-$\frac{3}{2}$ BEG model on a Bethe lattice was investigated by using the exact recursion equations [4]. The ferromagnetic spin-$\frac{3}{2}$ BEG model with repulsive biquadratic coupling, i.e., $K/J<0$, has been also investigated. An early attempt to study the ferromagnetic spin-$\frac{3}{2}$ BEG model was made by Barreto and Bonfim [5] and Bakkali et al. [6] within the mean-field approximation (MFA) and MC calculation, and the EFT, respectively. Barreto and Bonfim calculated only the phase diagrams for the isotropic spin-$\frac{3}{2}$ BEG model. Bakkali et al. also presented two phase diagrams, one for the spin-$\frac{3}{2}$ BC model and one for the isotropic spin-$\frac{3}{2}$ BEG model. Tucker [7] studied the spin-$\frac{3}{2}$ BEG model with $K/J<0$ by using the cluster variation method in pair approximation (CVMPA) and only presented the phase diagrams of the spin-$\frac{3}{2}$ BC and isotropic spin-$\frac{3}{2}$ BEG model for several values of the coordination number. Bakchich and Bouziani [8] calculate the phase diagram of the model only in $(T/J,D/J)$ for the two different values of K/J within an approximate RG approach of the Migdal–Kadanoff type. Recently, Ekiz et al. [9] investigated the model on Bethe lattice using the exact recursion equations and presented the phase diagrams in $(kT/J,K/J)$ plane for several values of D/J and in $(kT/J,D/J)$ plane for several values of K/J. Ekiz [10] extended the previous work, i.e. Ref. [9], for the presence of an external magnetic field. He considered only the ferromagnetic case.

On the other hand, as far as we know, multicritical behavior of the antiferromagnetic spin-$\frac{3}{2}$ BEG model has not been investigated whereas that of the antiferromagnetic spin-$\frac{3}{2}$ BC model was studied by Bakchich et al. [11] within the MFA, Bekhechi and Benyoussef [12] within the transfer matrix finite-size-scaling (TMFSS) calculations and MC simulations, Ekiz [13,14] by means of the exact recursion relations on Bethe lattice and by Keskin at al. [15] within the LACVM. These works [11–15] show that the antiferromagnetic spin-$\frac{3}{2}$ BC model exhibits a rich variety of behavior: second-order, first-order and critical points of different order. Therefore, the purpose of this work is to study multicritical behavior of the antiferromagnetic spin-$\frac{3}{2}$ BEG and to calculate the phase diagrams by using the LACVM which is identical to the MFA. Our recent works [15,16] display that the LACVM, in spite of its limitations such as the correlations of spin fluctuations not being considered, is an adequate starting point in which within this theoretical framework, it is easy to determine the complete phase diagrams. It also predicts the existence of the multicritical points. It is worthwhile to mention that the spin-$\frac{3}{2}$ BEG model with the J and K nearest-neighbor interactions was introduced to explain phase transition in $DyVO_4$ [17] qualitatively within the MFA by Sivardière and Blume [18]. Later, this model was used in a study of tricritical properties of a ternary fluid mixture and compared with the result of the experimental observations on the system of ethanol–water–carbon dioxide [19]. Moreover, the study of the antiferromagnetic phase is an important and actual area in magnetism.

The remainder of this work is organized as follows. In Section 2, we define the model briefly and obtain its solutions at equilibrium within the LACVM. Thermal variations of the order parameters are investigated in Section 3. In Section 4, transition temperatures are calculated precisely and the phase diagrams are presented in $(H/|J|,kT/|J|)$ plane. Section 5 contains the summary and conclusion.

2. Model and method

The spin-$\frac{3}{2}$ BEG model is defined as a two-sublattice model with spin variables $S_i=\pm\frac{3}{2},\pm\frac{1}{2}$ and $S_j=\pm\frac{3}{2},\pm\frac{1}{2}$ on sites of sublattices A and B, respectively. The average value of each of the spin states will be denoted by X_1^A, X_2^A, X_3^A and X_4^A on the sites of sublattice A and X_1^B, X_2^B, X_3^B and X_4^B on sublattice B, which are also called the state or point variables. X_1^A,X_1^B; X_2^A,X_2^B; X_3^A,X_3^B and X_4^A,X_4^B are the fractions of the spin values $+\frac{3}{2}$, $+\frac{1}{2}$, $-\frac{1}{2}$ and $-\frac{3}{2}$ on A and B sublattices, respectively. In order to account for the possible two-sublattice structure we need six long-range order parameters which are introduced as follows: $M_A\equiv\langle S_i^A\rangle$, $Q_A\equiv\langle (S_i^A)^2\rangle-\frac{5}{4}$, $R_A\equiv\frac{5}{3}\langle (S_i^A)^3\rangle-\frac{41}{12}\langle S_i^A\rangle$, $M_B\equiv\langle S_j^B\rangle$, $Q_B\equiv\langle (S_j^B)^2\rangle-\frac{5}{4}$, $R_B\equiv\frac{5}{3}\langle (S_j^B)^3\rangle-\frac{41}{12}\langle S_j^B\rangle$ for A and B sublattices, respectively. M_A and M_B are the average magnetizations which is the excess of one orientation over the other, called magnetizations; Q_A and Q_B are the quadrupolar moments which are the average squared magnetizations; and R_A and R_B are the octupolar-order parameters for A and B sublattices, respectively. The sublattice magnetizations define four different phases of the antiferromagnetic spin-$\frac{3}{2}$ BEG model: (i) The disordered (D) phase with $M_A=M_B>0$, (ii) the antiferromagnetic (AF) phase with $M_A=-M_B\neq 0$, (iii) the antiferrimagnetic (AI) phase with $M_A\neq -M_B\neq 0$ and (iv) the ferrimagnetic (I) phase with $M_A\neq M_B\neq 0$.