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Sentiment cycles in discrete-time homogeneous networks

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HIGHLIGHTS

- The paper approaches sentiment transition in a complex network.
- Agents are classified in neutral, optimists and pessimists.
- In continuous-time, the model delivers a stable steady-state outcome.
- In discrete-time, stability holds under a homogeneous network of degree one.
- Endogenous cycles emerge in discrete-time for a connectivity degree larger than one.

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ABSTRACT

Consider a network connecting individual agents that are endowed with distinct sentiments or 'views of the world'. Specifically, assume that each node in the network contains an agent that, at a given period t, can be found in one of five states: sentiment neutrality, exuberant optimism, non-exuberant optimism, exuberant pessimism and non-exuberant pessimism. Local interaction rules, similar to those one encounters in rumor propagation models, make agents change their sentiment as they contact with others. Under a continuous-time framework, the proposed setting delivers a stable fixed-point equilibrium, meaning that the shares of agents in each sentiment category will converge to constant steady-state levels. The inspection of the same structure of analysis in discrete-time indicates that the stability outcome continues to hold when the connectivity degree is equal to 1. However, this result might change as one considers higher-order connectivity. In this last case, persistent endogenous waves of optimism and pessimism emerge under a reasonable parameterization of the model.

1. Introduction

Human beings are commonly influenced by the contact they establish with those who occupy adjacent positions in a given social or economic network and, therefore, local interactions constitute a main driver of the micro choices and micro actions that potentially generate observable and meaningful macro patterns. Collective phenomena are typically perceived, as emphasized in Zschaler [1], as the macroscopic outcome emerging from the microscopic and decentralized interaction across a large number of individual units.

In this paper, a complex network is used as the benchmark structure to analyze how the local contact among individuals holding different 'views of the world' determines the overall evolution pattern of the sentiments of a given population.

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Complex networks are defined by Boccaletti et al. [2] as networks that possess an irregular and evolving structure. They are composed by thousands or millions of nodes that may or may not display different connectivity degrees.[1] Furthermore, the evolutionary nature of the network implies that agents placed at a given node will potentially change their status as they contact with agents in neighboring nodes.

In the framework under consideration, five categories of individuals will coexist at each time period: those with a neutral sentiment, but who are susceptible of becoming optimistic or pessimistic; non-exuberant optimists and pessimists; and exuberant optimists and pessimists. Local interaction among individuals with different sentiment status will be analyzed through the adaptation of the typical contact rules frequently considered in the analysis of rumor propagation or rumor spreading, as initially designed by Daley and Kendall [3] and Maki and Thompson [4].

Although rumor propagation in social networks is today a prolific line of research,[2] the adaptation of this theory to the field of sentiment formation and sentiment spreading has been scarce. A recent study, namely Zhao et al. [11], has launched the debate on the subject by proposing a model of sentiment contagion in complex networks, where individual agents face a binary selection mechanism. In their framework, depending on the model's parameterization, only one of two steady-state outcomes is feasible, i.e., in the long-term the entire population has adopted the optimistic view or, alternatively, it has become entirely pessimistic. Differently, in the current paper, the increased detail in terms of sentiment categories and interaction rules will conduct to a more generous set of potential long-term results: the dynamics will contemplate the possibility of a steady-state where part of the population remains optimistic, while others will stay in the pessimistic category; moreover, under certain conditions, the fixed-point steady-state will give place to endogenous fluctuations, that might be interpreted as representing waves of optimism and pessimism at the aggregate level.

The analysis to undertake starts by considering a general complex network in which agents interact locally and potentially change sentiments as the result of the contact they establish with each other. This network is first analyzed in continuous-time. The following results are obtained: (i) a fixed-point steady-state, where positive and constant densities of all sentiment categories persist over time, exists under explicit form and it is determinable; this result holds for a homogeneously mixing population and also for an inhomogeneous structure of interaction; (ii) circumscribing the analysis to the homogeneous network case, it is possible to prove that the steady-state corresponds to a stable equilibrium point.

The results described above are subject to change as one switches from a continuous-time to a discrete-time setting. By evaluating a discrete-time version of the dynamic system, exclusively in the context of homogeneous networks, the following conclusions are reached: (i) stability is confirmed, independently of parameter values, for a network of degree 1, i.e., for a network where all nodes are connected to a single neighboring node. The array of steady-state values is the same that was calculated in continuous-time; (ii) in networks of a higher connectivity degree, for which each node is connected to $k = 2, 3, \ldots$ other nodes, the fixed-point result is no longer pervasive; periodic and a-periodic cycles will repeatedly emerge. As a consequence, endogenous fluctuations in the time trajectories of the shares of neutral, optimistic and pessimistic agents are identified as a main result arising in this paper's sentiment-switching model.

The obtained cyclicality result is important because it constitutes a way of justifying how waves of optimism and pessimism eventually arise and persist in any possible economic or social context, as the mere consequence of a simple local interaction process. The only significant premise necessary to achieve this result is that one must avoid taking a homogeneous network of degree 1; a higher than 1 degree of connectivity is all that is required to arrive to the endogenous cycles' outcome. This is a useful result that can be applied, for instance, by economists to introduce an additional source of volatility in their reasoning about short-run macro behavior: animal spirits, in the form of alternating dominant optimism and dominant pessimism, emerge by the contact among neighboring agents that follow simple pre-specified rules of motion.

The remaining part of the paper is organized as follows. Section 2 is dedicated to a brief characterization of the structure of uncorrelated inhomogeneous and homogeneous networks and of the typical dynamics that occur within them. Section 3 introduces sentiment categories, thus giving a concrete substance to the assumed network structure and network relations. In Section 4, the steady-state of the sentiment model under continuous-time is derived and discussed; the respective stability is also evaluated. Section 5 approaches the dynamics under discrete-time and identifies the circumstances in which endogenous fluctuations are formed and sustained over time. Section 6 concludes.

2. Network dynamics

Consider a network of social relations, composed by a large number of nodes and by links that establish a connectivity pattern across nodes. The nodes represent individual agents, who can adopt distinct perspectives or interpretations about some socio-economic event. The network is undirected, in the sense that the links do not impose a pre-specified order through which an agent in one node exerts influence over another. The links provide the necessary tool for local interaction to occur; it is this interaction that allows each agent to eventually change her perspective or state, as she suffers the influence of an agent positioned in an adjacent or neighboring node.

[1] If every node in the network exhibits an identical connectivity degree, i.e., if there is a same number of links emanating from each node to the other nodes, the network is called homogeneous; otherwise, it acquires the designation of inhomogeneous.

[2] Some of the most representative studies published in this area in the last few years include Zanette [5], Thompson et al. [6], Nekovee et al. [7], Huo et al. [8], Zhao et al. [9] and Wang et al. [10].