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Empirical microeconomics action functionals

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HIGHLIGHTS

- The dynamics of commodity market prices is modeled by an action functional within the framework of statistical microeconomics.
- The correlation functions are investigated using a perturbation expansion in Feynman path integral and fitted to nine main commodities.
- The calibration results establish the existence of the action for commodity prices that was postulated to exit in Statistical microeconomics.

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ABSTRACT

A statistical generalization of microeconomics has been made in Baaquie (2013), where the market price of every traded commodity, at each instant of time, is considered to be an *independent random variable*. The dynamics of commodity market prices is modeled by an *action functional* – and the focus of this paper is to empirically determine the action functionals for different commodities. The correlation functions of the model are defined using a Feynman path integral. The model is calibrated using the unequal time correlation of the market commodity prices as well as their cubic and quartic moments using a perturbation expansion. The consistency of the perturbation expansion is verified by a numerical evaluation of the path integral. Nine commodities drawn from the energy, metal and grain sectors are studied and their market behavior is described by the model to an accuracy of over 90% using only six parameters. The paper *empirically* establishes the existence of the action functional for commodity prices that was *postulated* to exist in Baaquie (2013).

1. Introduction

Supply and demand are inseparable and the view taken in statistical microeconomics [1] is that supply and demand are two facets of a single entity, namely the microeconomics *potential function* $\mathcal{V}[\mathbf{p}]$. The potential is chosen to be the sum of supply and demand, namely [1]

$$\mathcal{V}[\mathbf{p}] = \mathcal{D}[\mathbf{p}] + \mathcal{S}[\mathbf{p}]. \tag{1}$$

The potential function $\mathcal{V}[\mathbf{p}]$, similar to mechanics, drives the evolution of market prices. For the special case when the prices are constant (time independent) – given by the constant prices $\mathbf{p}_0 = (p_{01}, p_{02}, .., p_{0N})$ – the prices *minimize the value* of the potential; namely $\mathcal{V}[\mathbf{p}_0]$ is a minimum of $\mathcal{V}[\mathbf{p}]$.

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The break up of the microeconomics potential into a supply and demand piece need not hold in general for all values of the price since the break up is essentially an *asymptotic property* of the microeconomics potential. One expects from the behavior of consumers and producers that the demand for a commodity increases with decreasing price and concomitantly, the production of a commodity increases with increasing price. Hence, the most general microeconomics potential is stipulated to have the following two limiting cases

$$\mathcal{V}[\mathbf{p}] \simeq \begin{cases} \mathcal{D}[\mathbf{p}] : p_i \to 0 \\ \mathcal{S}[\mathbf{p}] : p_i \to \infty. \end{cases} \tag{2}$$

In the framework of statistical microeconomics, stationary prices are determined by the minimum value of the microeconomics potential, which replaces the standard microeconomics procedure of setting supply equal to demand.

The dynamics of market prices is determined by assigning a **joint probability distribution** for all possible evolutions of the stochastic market prices. The probability of the stochastic evolution of market prices is *postulated* to be proportional to the Boltzmann distribution, namely

$$\text{Joint probability distribution} \propto \exp\{-\mathcal{A}[\mathbf{p}]\} \tag{3}$$

where the action functional $\mathcal{A}[\mathbf{p}]$ determines the likelihood of the evolution of all the different values taken by all the prices.

In analogy with mechanics, the action functional is taken to be the sum of the potential term $\mathcal{V}[p]$ with a *kinetic term* $\mathcal{T}$, namely

$$\mathcal{A}[\mathbf{p}] = \int_{-\infty}^{+\infty} dt \mathcal{L}(t) = \int_{-\infty}^{+\infty} dt \Big(\mathcal{T}[\mathbf{p}(t)] + \mathcal{V}[\mathbf{p}(t)]\Big). \tag{4}$$

The action functional $\mathcal{A}[\mathbf{p}]$ depends on the *function* $\mathbf{p}(t)$, $t \in [-\infty, +\infty]$: each possible function $p(t)$ gives one numerical value for $\mathcal{A}[\mathbf{p}]$. For this reason $\mathcal{A}[\mathbf{p}]$ is called a functional of the price function and is called the action functional, or action in brief.

The Lagrangian given by

$$\mathcal{L}(t) = \mathcal{T}[\mathbf{p}(t)] + \mathcal{V}[\mathbf{p}(t)]. \tag{5}$$

The kinetic terms $\mathcal{T}[\mathbf{p}(t)]$ contains the time derivatives of the prices and together with the potential function, determines the time dependence of the stochastic prices; in particular, $\exp\{-\mathcal{A}[\mathbf{p}]\}$ determines the likelihood of the different random trajectories of the random prices.

2. Model of the microeconomics potential

The demand function is modeled to be [1]

$$\mathcal{D}[\mathbf{p}] = m \sum_{i=1}^{N} \frac{d_i}{p_i^{a_i}}; \quad a_i,\ d_i > 0 \tag{6}$$

and the supply function is modeled to be [1]

$$\mathcal{S}[\mathbf{p}] = m \sum_{i=1}^{N} s_i p_i^{b_i}; \quad b_i,\ s_i > 0. \tag{7}$$

The coefficients d_i, s_i, according to Ref. [2], are determined by macroeconomic factors such as interest rates, unemployment, inflation and so on. In the statistical microeconomics model, the coefficients d_i, s_i are determined from the historical prices of a commodity. It is our view that all the macroeconomic information that affects a commodity is contained in its price. Hence it is a consistency check to see if the values of d_i, s_i given by a macroeconomic analysis of a commodity agree with the result obtained by studying solely the price of a commodity.

The sum of the demand and supply function yields the microeconomics potential

$$\begin{aligned} \mathcal{V}[\mathbf{p}] &= \mathcal{D}[\mathbf{p}] + \mathcal{S}[\mathbf{p}] \\ &= m\left[\sum_{i=1}^{N} \frac{d_i}{p_i^{a_i}} + \sum_{i=1}^{N} s_i p_i^{b_i}\right]; \quad d_i,\ s_i > 0;\ a, b > 0. \end{aligned} \tag{8}$$

The model microeconomics potential has the following expected asymptotic behavior of exhibiting a supply and demand function as expected from Eq. (2)

$$\mathcal{V}[\mathbf{p}] \simeq \begin{cases} \mathcal{D}[\mathbf{p}] = \sum_{i=1}^{N} \frac{d_i}{p_i^{a_i}}; \quad p_i \to 0 \\ \mathcal{S}[\mathbf{p}] = \sum_{i=1}^{N} s_i p_i^{b_i}; \quad p_i \to \infty. \end{cases}$$