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Synchronization of fractional order complex dynamical networks

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HIGHLIGHTS

- We proposed a fractional order controller and a synchronization law for fractional order complex networks.
- Some new sufficient synchronization criteria are proposed.
- These criteria can apply to an arbitrary fractional order complex network.

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ABSTRACT

In this letter the synchronization of complex dynamical networks with fractional order chaotic nodes is studied. A fractional order controller for synchronization of complex network is presented. Some new sufficient synchronization criteria are proposed based on the Lyapunov stability theory and the LaSalle invariance principle. These synchronization criteria can apply to an arbitrary fractional order complex network in which the coupling-configuration matrix and the inner-coupling matrix are not assumed to be symmetric or irreducible. It means that this method is more general and effective. Numerical simulations of two fractional order complex networks demonstrate the universality and the effectiveness of the proposed method.

1. Introduction

Many large-scale systems in nature and human societies, such as ecosystems, biological neural networks, the Internet, metabolic pathways, the WWW, and electrical power grids, can be described by networks with the nodes representing individuals in the systems and the edges representing the connections among them [1–3]. In recent years, the study of various complex networks has attracted increasing attention from researchers in various fields of physics, engineering, mathematics, sociology, and biology [4–6].

On the basis of complex network models, the dynamics of complex networks have been extensively investigated, with special emphasis on the interplay between the complexity in the overall topology and the local dynamical properties of the coupled nodes. Among various dynamics, synchronization of complex network is a typical collective behavior in nature. Therefore, a large amount of work has been devoted to the study of synchronization in various large-scale complex networks [7–14]. The authors investigated the problem of controllability of a realistic neuronal network of the cat under constraints on control gains by utilizing a MDyHF [8]. Local synchronization was investigated by the transverse stability to the synchronization manifold, where synchronization was discussed on small-world and scale-free networks in

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Refs. [9,10]. In Ref. [11], a distance from the collective states to the synchronization manifold was defined, based on which some results were obtained for global synchronization of coupled systems. A general criterion was given in Ref. [12], where the network sizes can be extended to be much larger compared to those very small ones studied in Ref. [13]. The paper deals with the problem of robust adaptive synchronization of dynamical networks with stochastic coupling by means of evolutionary algorithms [14].

However, it should be noted that most of the studies are mainly concentrated on the integer-order complex networks. In recent years, fractional calculus, as a generalization of ordinary differentiation and integration, has received much attention due to its application in physics and engineering [15,16]. Many systems were found in interdisciplinary fields, such as dielectric polarization, viscoelasticity, quantum evolution of complex system, electromagnetic wave, and fractional differential equations [17–19]. These research efforts have shown that fractional derivatives provide an excellent tool for describing the memory and hereditary properties of various materials and processes. By means of fractional calculus theory, there are many fractional order dynamical systems, such as the fractional order Lorenz system [20], the fractional order Chua system [21], the fractional-order Chen system [22] and so on. In the real world, many systems usually consist of a large number of highly interconnected fractional order dynamical units to form the complex networks. Therefore, it is essential to study synchronization of the fractional order complex dynamical networks.

Even though synchronization of the integer-order complex network has been intensively studied by various control schemes in the past few years, unfortunately research on the synchronization of fractional order complex dynamical network has received less attention in spite of its practical significance. To our knowledge, Wang and Zhang studied the synchronized motions in a star-shaped network of coupled fractional order systems [23]. In Ref. [24], the authors pioneered in addressing the pining control problem of fractional-order weighted complex dynamical networks. In Ref. [25], the outer synchronization between two different fractional order general complex dynamical networks was investigated by applying the nonlinear control to all nodes, which leads to excessive control costs. The synchronized motions in N-coupled incommensurate fractional-order chaotic systems with ring connection was studied in Ref. [26]. The outer synchronization between two coupled complex networks with fractional-order dynamics was studied by an openplus-closed-loop scheme [27]. The authors studied the synchronization and anti-synchronization behavior between integer-order dynamical networks and fractional-order dynamical systems via a Takagi–Sugeno fuzzy model in Ref. [28].

Hence, in this paper, we present a fractional order controller for the synchronization of fractional order complex network. Some sufficient synchronization criteria are proposed based on the Lyapunov stability theory and the LaSalle invariance principle. This method can apply to an arbitrary fractional order complex networks in which the coupling-configuration matrices and the inner-coupling matrices are not assumed to be symmetric and irreducible. So this method is more general and effective than others. Numerical simulations of two fractional order complex networks demonstrate the universality and the effectiveness of the proposed method.

2. Fractional derivatives and fractional order complex dynamical network

2.1. Fractional derivatives

The fractional calculus plays an important role in modern science. In this paper we mainly use the Caputo fractional operators [15,16,29]. The Caputo definition of the fractional derivative, which sometimes called smooth fractional derivative, is described as

$$\mathrm{D}_t^{\alpha} = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t (t-\tau)^{m-\alpha-1} f^{(m)}(\tau)\mathrm{d}\tau, & m-1<\alpha<m, \\ \dfrac{\mathrm{d}^m}{\mathrm{d}t^m} f(t), & \alpha = m, \end{cases} \tag{1}$$

where m is the lowest integer which is not less than α, and Γ is the Gamma function,

$$\Gamma(z) = \int_0^{\infty} t^{z-1}\mathrm{e}^{-t}\mathrm{d}t. \tag{2}$$

In this paper we mainly consider the order $0 < \alpha < 1$. There are some general properties of the fractional-order derivative which are described as follows [15,16].

Property 1. *Caputo fractional derivative is a linear operator, i.e.,*

$$\mathrm{D}_t^{\alpha}(\lambda f(t) + \mu g(t)) = \lambda \mathrm{D}_t^{\alpha} f(t) + \mu \mathrm{D}_t^{\alpha} g(t), \tag{3}$$

where λ, μ are real constants.

Property 2. *Caputo fractional derivative satisfies additive index law (semigroup property), i.e.*

$$\mathrm{D}_t^{\alpha}\mathrm{D}_t^{\beta} f(t) = \mathrm{D}_t^{\beta}\mathrm{D}_t^{\alpha} f(t) = \mathrm{D}_t^{\alpha+\beta} f(t), \tag{4}$$

holds under some reasonable constraints on the function $f(t)$.